اختبار نهاية الوحدة صفحة 105	
1	$\int_0^2 e^{2x} = \frac{1}{2}e^{2x}\Big\|_0^2 = \frac{1}{2}e^4 - \frac{1}{2} \ldots\ldots\ldots (d)$
2	$\int_{-4}^{4}(4-\|x\|)\,dx = \int_{-4}^{0}(4+x)\,dx + \int_0^4 (4-x)\,dx$ $= \left(4x+\frac{1}{2}x^2\right)\Big\|_{-4}^{0} + \left(4x-\frac{1}{2}x^2\right)\Big\|_0^4$ $= -(-16+8)+(16-8)$ $= 16 \ldots\ldots\ldots\ldots\ldots (c)$
3	$A = \int_{-1}^{2}\left(x^3-3x^2+4-(x^2-x-2)\right)dx$ $= \int_{-1}^{2}(x^3-4x^2+x+6)\,dx \ldots\ldots\ldots\ldots (a)$
4	$\int \frac{dy}{y} = \int 2x\,dx \Rightarrow \ln\|y\| = x^2 + C$ $(0,1) \Rightarrow 0 = 0 + C \Rightarrow C = 0$ $\Rightarrow \ln\|y\| = x^2 \Rightarrow \|y\| = e^{x^2}$ ولكن $y = -e^{x^2}$ لا يحقق النقطة $(0,1)$، إذن، الحل هو $y = e^{x^2}$ ....(a)
5	$\int \frac{1}{\sqrt{e^x}}dx = \int e^{-\frac{1}{2}x}dx = -2e^{-\frac{1}{2}x} + C$
6	$\int\left(\tan 2x + e^{3x} - \frac{1}{x}\right)dx = \int\left(-\frac{1}{2}\times\frac{-2\sin 2x}{\cos 2x} + e^{3x} - \frac{1}{x}\right)dx$ $= -\frac{1}{2}\ln\|\cos 2x\| + \frac{1}{3}e^{3x} - \ln\|x\| + C$

7	$\int \csc^2 x\,(1+\tan^2 x)dx = \int\left(\csc^2 x + \frac{1}{\sin^2 x}\times\frac{\sin^2 x}{\cos^2 x}\right)dx$ $= \int (\csc^2 x + \sec^2 x)dx$ $= -\cot x + \tan x + C$
8	$\int \frac{e^{2x}}{e^{2x}+5}dx = \frac{1}{2}\int\frac{2e^{2x}}{e^{2x}+5}dx = \frac{1}{2}\ln(e^{2x}+5)+C$
9	$\int \frac{2x^2+7x-3}{x-2}dx = \int\left(2x+11+\frac{19}{x-2}\right)dx$ $= x^2+11x+19\ln\lvert x-2\rvert + C$
10	$\int \sec^2(2x-1)dx = \frac{1}{2}\tan(2x-1)+C$
11	$\int \cot(5x+1)dx = \frac{1}{5}\int\frac{5\cos(5x+1)}{\sin(5x+1)}dx = \frac{1}{5}\ln\lvert\sin(5x+1)\rvert + C$
12	$\int_0^{\frac{\pi}{2}} \sin x\cos x = \frac{1}{2}\int_0^{\frac{\pi}{2}}\sin 2x = -\frac{1}{4}\cos 2x\Big\rvert_0^{\frac{\pi}{2}} = -\frac{1}{4}(-1-1) = \frac{1}{2}$
13	$\int_0^{\pi} \cos^2\frac{1}{2}x\ dx = \frac{1}{2}\int_0^{\pi}(1+\cos x)dx$ $= \frac{1}{2}(x+\sin x)\rvert_0^{\pi} = \frac{1}{2}\big((\pi)+(0)\big)-0 = \frac{\pi}{2}$
14	$\int_0^2 \lvert x^3-1\rvert\, dx = \int_0^1 (1-x^3)\,dx + \int_1^2 (x^3-1)\,dx$ $= \left(x-\frac{1}{4}x^4\right)\Big\rvert_0^1 + \left(\frac{1}{4}x^4 - x\right)\Big\rvert_1^2 = \left(\frac{3}{4}\right)+\left(4-2+\frac{3}{4}\right) = \frac{7}{2}$
15	$\int_0^{\frac{\pi}{4}}(\sec^2 x+\cos 4x)\,dx = \left(\tan x + \frac{1}{4}\sin 4x\right)\Big\rvert_0^{\frac{\pi}{4}} = (1)-(0) = 1$

16	$\int_0^{\frac{\pi}{3}} \left(\sin\left(2x+\frac{\pi}{3}\right) - 1 + \cos 2x\right)\, dx$ $= \left(-\frac{1}{2}\cos\left(2x+\frac{\pi}{3}\right) - x + \frac{1}{2}\sin 2x\right)\Big|_0^{\frac{\pi}{3}} = \left(\frac{1}{2} - \frac{\pi}{3} + \frac{\sqrt{3}}{4}\right) - \left(-\frac{1}{4}\right)$ $= \frac{3+\sqrt{3}}{4} - \frac{\pi}{3}$
17	$\int_0^{\frac{\pi}{8}} \sin 2x \cos 2x\, dx = \frac{1}{2}\int_0^{\frac{\pi}{8}} \sin 4x\, dx = -\frac{1}{8}\cos 4x\Big|_0^{\frac{\pi}{8}} = 0 - \left(-\frac{1}{8}\right) = \frac{1}{8}$
18	$\int \frac{4}{x^2-4}dx = \int \frac{4}{(x-2)(x+2)}dx$ $\frac{4}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$ $A(x+2) + B(x-2) = 4$ $x = 2 \Rightarrow A = 1$ $x = -2 \Rightarrow B = -1$ $\frac{4}{(x-2)(x+2)} = \frac{1}{x-2} + \frac{-1}{x+2}$ $\int \frac{4}{x^2-4}dx = \int \frac{4}{(x-2)(x+2)}dx$ $= \int \left(\frac{1}{x-2} + \frac{-1}{x+2}\right)dx$ $= \ln\lvert x-2\rvert - \ln\lvert x+2\rvert + C$ $= \ln\left\lvert\frac{x-2}{x+2}\right\rvert + C$

19	$\int \frac{x+7}{x^2-x-6}dx = \int \frac{x+7}{(x-3)(x+2)}dx$ $\frac{x+7}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$ $A(x+2)+B(x-3) = x+7$ $x=-2 \Rightarrow B=-1$ $x=3 \Rightarrow A=2$ $\frac{x+7}{(x-3)(x+2)} = \frac{2}{x-3} + \frac{-1}{x+2}$ $\int \frac{x+7}{x^2-x-6}dx = \int \frac{x+7}{(x-3)(x+2)}dx$ $= \int \left(\frac{2}{x-3} + \frac{-1}{x+2}\right)dx$ $= 2\ln\|x-3\| - \ln\|x+2\| + C$
20	$\int \frac{x-1}{x^2-2x-8}dx = \frac{1}{2}\int \frac{2(x-1)}{x^2-2x-8}dx = \frac{1}{2}\ln\|x^2-2x-8\| + C$

21	$\int \frac{x^2+3}{x^3+x}dx = \int \frac{x^2+3}{x(x^2+1)}dx$ $\frac{x^2+3}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ $A(x^2+1) + (Bx+C)(x) = x^2+3$ $x = 0 \Rightarrow A = 3$ $x = 1 \Rightarrow 2A + B + C = 4 \Rightarrow B + C = -2$ $x = -1 \Rightarrow 2A + B - C = 4 \Rightarrow B - C = -2$ $\Rightarrow B = -2 \ , C = 0$ $\frac{x^2+3}{x(x^2+1)} = \frac{3}{x} + \frac{-2x}{x^2+1}$ $\int \frac{x^2+3}{x^3+x}dx = \int \frac{x^2+3}{x(x^2+1)}dx$ $= \int \left(\frac{3}{x} + \frac{-2x}{x^2+1}\right)dx$ $= 3\ln\lvert x\rvert - \ln\lvert x^2+1\rvert + C = \ln\left\lvert\frac{x^3}{x^2+1}\right\rvert + C$
22	$\frac{1}{x^2(1-x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{1-x}$ $Ax(1-x) + B(1-x) + C(x^2) = 1$ $x = 0 \Rightarrow B = 1$ $x = 1 \Rightarrow C = 1$ $x = -1 \Rightarrow -2A + 2B + C = 1 \Rightarrow A = 1$ $\frac{1}{x^2(1-x)} = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{1-x}$ $\int \frac{1}{x^2(1-x)}dx = \int \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{1-x}\right)dx$ $= \ln\lvert x\rvert - \frac{1}{x} - \ln\lvert 1-x\rvert + C$ $= \ln\left\lvert\frac{x}{1-x}\right\rvert - \frac{1}{x} + C$

23	$u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$ $\int \frac{\sin x}{\cos^2 x - 3\cos x} = \int \frac{\sin x}{u^2 - 3u} \times \frac{du}{-\sin x} = \int \frac{1}{3u - u^2} du$ $\frac{1}{3u - u^2} = \frac{1}{u(3-u)} = \frac{A}{u} + \frac{B}{3-u}$ $\Rightarrow A(3-u) + Bu = 1$ $u = 0 \quad \Rightarrow A = \frac{1}{3}$ $u = 3 \quad \Rightarrow B = \frac{1}{3}$ $\int \frac{1}{3u - u^2} du = \int \left(\frac{\frac{1}{3}}{u} + \frac{\frac{1}{3}}{3-u} \right) du$ $= \frac{1}{3}\ln\lvert u\rvert - \frac{1}{3}\ln\lvert 3-u\rvert + C$ $= \frac{1}{3}\ln\left\lvert \frac{\cos x}{3 - \cos x} \right\rvert + C$
24	$u = \sqrt{x} \Rightarrow u^2 = x, \quad dx = 2u\, du$ $\int \frac{\sqrt{x}}{x-4} = \int \frac{u}{u^2-4} \times 2u\, du = \int \frac{2u^2}{u^2-4} du = \int \left(2 + \frac{8}{u^2-4}\right) du$ $\frac{8}{u^2-4} = \frac{A}{u-2} + \frac{B}{u+2}$ $\Rightarrow A(u+2) + B(u-2) = 8$ $u = 2 \quad \Rightarrow A = 2$ $u = -2 \quad \Rightarrow B = -2$ $\int \frac{\sqrt{x}}{x-4} = \int \left(2 + \frac{2}{u-2} + \frac{-2}{u+2}\right) du$ $= 2u + 2\ln\lvert u-2\rvert - 2\ln\lvert u+2\rvert + C$ $= 2\sqrt{x} + 2\ln\left\lvert \frac{\sqrt{x}-2}{\sqrt{x}+2} \right\rvert + C$

25	$u = 1 + \tan x \;\Rightarrow dx = \dfrac{du}{\sec^2 x}$ $\int \sec^2 x \tan x \sqrt{1+\tan x}\, dx = \int \sec^2 x (u-1)\sqrt{u}\, \dfrac{du}{\sec^2 x}$ $= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$ $= \dfrac{2}{5}u^{\frac{5}{2}} - \dfrac{2}{3}u^{\frac{3}{2}} + C$ $= \dfrac{2}{5}(1+\tan x)^{\frac{5}{2}} - \dfrac{2}{3}(1+\tan x)^{\frac{3}{2}} + C$
26	$u = 4 - 3x \;\Rightarrow dx = \dfrac{du}{-3} \;, x = \dfrac{4-u}{3}$ $\int \dfrac{x}{\sqrt[3]{4-3x}}\, dx = \int \dfrac{\frac{1}{3}(4-u)}{u^{\frac{1}{3}}} \times \dfrac{du}{-3} = -\dfrac{1}{9}\int \left(4u^{-\frac{1}{3}} - u^{\frac{2}{3}}\right) du$ $= -\dfrac{1}{9}\left(6u^{\frac{2}{3}} - \dfrac{3}{5}u^{\frac{5}{3}}\right) + C = -\dfrac{2}{3}u^{\frac{2}{3}} + \dfrac{1}{15}u^{\frac{5}{3}} + C$ $= -\dfrac{2}{3}(4-3x)^{\frac{2}{3}} + \dfrac{1}{15}(4-3x)^{\frac{5}{3}} + C$ ملاحظة: يمكن حل هذا التكامل بالأجزاء أيضًا
27	$u = \ln x \Rightarrow \dfrac{du}{dx} = \dfrac{1}{x} \;, dx = x\, du$ $\int \dfrac{(\ln x)^6}{x}\, dx = \int \dfrac{x u^6}{x}\, du = \int u^6 du = \dfrac{1}{7}u^7 + C = \dfrac{1}{7}(\ln x)^7 + C$

28	$u = x - 2 \Rightarrow x = u + 2 \ , dx = du$ $\int (x+1)^2\sqrt{x-2}\,dx = \int (u+3)^2 u^{\frac{1}{2}}\,du$ $= \int (u^2+6u+9)u^{\frac{1}{2}}\,du = \int \left(u^{\frac{5}{2}} + 6u^{\frac{3}{2}} + 9u^{\frac{1}{2}}\right) du$ $= \frac{2}{7}u^{\frac{7}{2}} + \frac{12}{5}u^{\frac{5}{2}} + 6u^{\frac{3}{2}} + C$ $= \frac{2}{7}(x-2)^{\frac{7}{2}} + \frac{12}{5}(x-2)^{\frac{5}{2}} + 6(x-2)^{\frac{3}{2}} + C$ ملاحظة: يمكن حل هذا التكامل بالأجزاء مرتين
29	$\int x\csc^2 x\,dx$ $u = x \qquad dv = \csc^2 x\,dx$ $du = dx \qquad v = -\cot x$ $\int x\csc^2 x\,dx = -x\cot x + \int \cot x\,dx = -x\cot x + \int \frac{\cos x}{\sin x}\,dx$ $= -x\cot x + \ln\lvert\sin x\rvert + C$
30	$u = x^2 - 5x \qquad dv = e^x\,dx$ $du = (2x-5)dx \qquad v = e^x$ $\int (x^2-5x)e^x\,dx = (x^2-5x)e^x - \int (2x-5)e^x\,dx$ $u = 2x - 5 \qquad dv = e^x\,dx$ $du = 2dx \qquad v = e^x$ $\int (2x-5)e^x\,dx = (2x-5)e^x - \int 2e^x\,dx$ $= (2x-5)e^x - 2e^x + C$ $\int (x^2-5x)e^x\,dx = (x^2-5x)e^x - (2x-5)e^x + 2e^x + C$ $= e^x(x^2 - 7x + 7) + C$

31	$u = x \qquad dv = \sin 2x\ dx$ $du = dx \qquad v = -\frac{1}{2}\cos 2x$ $\int x \sin 2x\ dx = -\frac{1}{2}x\cos 2x - \int -\frac{1}{2}\cos 2x\ dx$ $= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$
32	$u = t^2 \Rightarrow dt = \frac{du}{2t}$ $t = 0 \Rightarrow u = 0$ $t = 1 \Rightarrow u = 1$ $\int_0^1 t\, 3^{t^2}\, dt = \int_0^1 t3^u \frac{du}{2t} = \frac{1}{2}\int_0^1 3^u du = \frac{3^u}{2\ln 3}\Big\|_0^1$ $= \frac{3}{2\ln 3} - \frac{1}{2\ln 3} = \frac{1}{\ln 3}$

33	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^3 x\, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x\,(\csc^2 x - 1)\, dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x \csc^2 x\, dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x\, dx$ $u = \cot x \Rightarrow dx = \frac{du}{-\csc^2 x}$ $x = \frac{\pi}{4} \Rightarrow u = 1$ $x = \frac{\pi}{3} \Rightarrow u = \frac{1}{\sqrt{3}}$ $\Rightarrow \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x \csc^2 x\, dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x\, dx = \int_{1}^{\frac{1}{\sqrt{3}}} -u\, du - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos x}{\sin x}\, dx$ $= -\frac{1}{2}u^2\Big\|_{1}^{\frac{1}{\sqrt{3}}} - \ln\|\sin x\|\Big\|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= -\frac{1}{2}\left(\frac{1}{3} - 1\right) - (\ln\frac{\sqrt{3}}{2} - \ln\frac{\sqrt{2}}{2})$ $= \frac{1}{3} - \ln\frac{\sqrt{3}}{\sqrt{2}}$ $= \frac{1}{3} - \frac{1}{2}\ln\frac{3}{2}$
34	$u = 4 + 3\sin x \Rightarrow dx = \frac{du}{3\cos x}$ $x = -\pi \Rightarrow u = 4$ $x = \pi \Rightarrow u = 4$ $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4 + 3\sin x}} dx = \int_{4}^{4} \frac{\cos x}{\sqrt{u}} \times \frac{du}{3\cos x} = \frac{1}{3}\int_{4}^{4} \frac{du}{\sqrt{u}} = 0$

35	$\int_{-1}^{0}\frac{x^2-x}{x^2+x-2}dx=\int_{-1}^{0}\frac{x(x-1)}{(x-1)(x+2)}dx$ $=\int_{-1}^{0}\frac{x}{x+2}dx=\int_{-1}^{0}\left(1-\frac{2}{x+2}\right)dx$ $=(x-2\ln\|x+2\|)\|_{-1}^{0}=0-2\ln 2-(-1-2\ln 1)=1-2\ln 2$
36	$\frac{32x^2+4}{16x^2-1}=2+\frac{6}{16x^2-1}=2+\frac{A}{4x-1}+\frac{B}{4x+1}$ $\Rightarrow A(4x+1)+B(4x-1)=6$ $x=\frac{1}{4}\ \Rightarrow A=3$ $x=-\frac{1}{4}\ \Rightarrow B=-3$ $\int_{1}^{2}\frac{32x^2+4}{16x^2-1}dx=\int_{1}^{2}\left(2+\frac{3}{4x-1}+\frac{-3}{4x+1}\right)dx$ $=\left(2x+\frac{3}{4}\ln\|4x-1\|-\frac{3}{4}\ln\|4x+1\|\right)\Big\|_{1}^{2}$ $=\left(4+\frac{3}{4}\ln 7-\frac{3}{4}\ln 9\right)-\left(2+\frac{3}{4}\ln 3-\frac{3}{4}\ln 5\right)$ $=2+\frac{3}{4}\ln\frac{35}{27}$
37	$u=\ln 2x\qquad dv=x\,dx$ $du=\frac{1}{x}\qquad v=\frac{x^2}{2}$ $\int_{\frac{1}{2}}^{\frac{e}{2}}x\ln 2x\ dx=\frac{x^2}{2}\ln 2x\Big\|_{\frac{1}{2}}^{\frac{e}{2}}-\int_{\frac{1}{2}}^{\frac{e}{2}}\frac{x}{2}\ dx$ $=\frac{x^2}{2}\ln 2x\Big\|_{\frac{1}{2}}^{\frac{e}{2}}-\frac{1}{4}x^2\Big\|_{\frac{1}{2}}^{\frac{e}{2}}$ $=\frac{1}{16}(e^2+1)$

38	$s(10)-s(0)=\int_0^{10} v(t)dt = R_1 - R_2 + R_3$ $=\frac{1}{2}(2)(4)-\frac{1}{2}(2)(4)+\frac{1}{2}(3+6)(4)=18 \text{ m}$
39	$\int_0^{10} \lvert v(t)\rvert dt = R_1 + R_2 + R_3 = 4+4+18 = 26 \text{ m}$
40	$s(10)-s(0)=18 \Rightarrow s(10)-0=18 \Rightarrow s(10)=18 \text{ m}$
41	$x^2 = x^{\frac{1}{2}} \Rightarrow x^4 = x \Rightarrow x^4 - x = 0$ $\Rightarrow x(x^3-1)=0 \Rightarrow x=0, \quad x=1$ $A=\int_0^1 (\sqrt{x}-x^2)dx = \left(\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{3}x^3\right)\Big\|_0^1 = \left(\frac{2}{3}-\frac{1}{3}\right)-(0)=\frac{1}{3}$
42	$x = x^3 \Rightarrow x(x^2-1)=0 \Rightarrow x=0, \quad x=1, \quad x=-1$ $A=\int_{-1}^{0}(x^3-x)dx+\int_0^1 (x-x^3)dx$ $=\left(\frac{1}{4}x^4-\frac{1}{2}x^2\right)\Big\|_{-1}^{0}+\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)\Big\|_0^1=\frac{1}{2}$
43	$x^2+2=-x \Rightarrow x^2+x+2=0$ هذه المعادلة التربيعية لا حلول لها، لأن المميز سالب، إذن، منحنيا الاقترانين لا يتقاطعان. $A=\int_{-2}^{2}(x^2+2+x)dx=\left(\frac{1}{3}x^3+2x+\frac{1}{2}x^2\right)\Big\|_{-2}^{2}=\frac{40}{3}$

44	$\frac{x^2}{x^2-1} = 1 + \frac{1}{x^2-1} = 1 + \frac{A}{x-1} + \frac{B}{x+1}$ $\Rightarrow A(x+1) + B(x-1) = 1$ $x = 1 \Rightarrow A = \frac{1}{2}$ $x = -1 \Rightarrow B = -\frac{1}{2}$ $\int_2^5 \frac{x^2}{x^2-1} dx = \int_2^5 \left(1 + \frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}}{x+1}\right) dx$ $= \left(x + \frac{1}{2}\ln\lvert x-1\rvert - \frac{1}{2}\ln\lvert x+1\rvert\right)\Big\|_2^5 = 3 + \frac{1}{2}\ln 2$
45	$D = \int_1^{10} v(t)dt = \int_1^{10} \left(\frac{1}{9}t - (t+6)^{-\frac{1}{2}}\right) dt$ $= \left(\frac{1}{18}t^2 - 2\sqrt{t+6}\right)\Big\|_1^{10} = \left(2\sqrt{7} - \frac{5}{2}\right) \text{m} \approx 2.792 \text{ m}$
46	$v(t) = \frac{1}{9}t - (t+6)^{-\frac{1}{2}}$ لتكن d المسافة المقطوعة وهي تمثل المساحة بين منحنى $\lvert v(t)\rvert$ والمحور t بين المستقيمين $t = 1, \quad t = 10$ $d = \int_1^{10} \lvert v(t)\rvert dt = \int_1^{10} \left\lvert \frac{1}{9}t - (t+6)^{-\frac{1}{2}} \right\rvert dt$ $\frac{1}{9}t - (t+6)^{-\frac{1}{2}} = 0 \Rightarrow \frac{t}{9} = \frac{1}{\sqrt{t+6}} \Rightarrow t\sqrt{t+6} = 9 \Rightarrow t^2(t+6) = 81$ $\Rightarrow t^3 + 6t^2 - 81 = 0 \Rightarrow (t-3)(t^2+9t+81) = 0 \Rightarrow t = 3$ $\Rightarrow d = -\int_1^3 \left(\frac{1}{9}t - (t+6)^{-\frac{1}{2}}\right) dt + \int_3^{10} \left(\frac{1}{9}t - (t+6)^{-\frac{1}{2}}\right) dt$ $= \left(2\sqrt{t+6} - \frac{1}{18}t^2\right)\Big\|_1^3 + \left(\frac{1}{18}t^2 - 2\sqrt{t+6}\right)\Big\|_3^{10} = \frac{155}{18} - 2\sqrt{7} \approx 3.32 \text{ m}$

47	$(1+\sin 2x)^2 = 0 \Rightarrow \sin 2x = -1$ $2x = \frac{3\pi}{2} \Rightarrow x = \frac{3\pi}{4}$ هذا هو أول حل موجب للمعادلة $\Rightarrow A\left(\frac{3\pi}{4}, 0\right)$
48	$A(R) = \int_0^{\frac{3\pi}{4}} (1+\sin 2x)^2 dx = \int_0^{\frac{3\pi}{4}} (1 + 2\sin 2x + \sin^2 2x) dx$ $= \int_0^{\frac{3\pi}{4}} \left(1 + 2\sin 2x + \frac{1}{2}(1 - \cos 4x)\right) dx$ $= \int_0^{\frac{3\pi}{4}} \left(\frac{3}{2} + 2\sin 2x - \frac{1}{2}\cos 4x\right) dx$ $= \left(\frac{3}{2}x - \cos 2x - \frac{1}{8}\sin 4x\right)\Big\|_0^{\frac{3\pi}{4}} = \frac{9\pi}{8} + 1$
49	$x^2 \ln 2x = 0 \Rightarrow x = 0$ يهمل لأنه خارج مجال اقتران اللوغاريتم $\ln 2x = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$ $A = \int_{\frac{1}{2}}^{1} x^2 \ln 2x \, dx$ $u = \ln 2x \qquad dv = x^2\, dx$ $du = \frac{1}{x}\, dx \qquad v = \frac{1}{3}x^3$ $\int x^2 \ln 2x\, dx = \frac{1}{3}x^3 \ln 2x - \int \frac{1}{3}x^3 \frac{1}{x} dx$ $= \frac{1}{3}x^3 \ln 2x - \frac{1}{9}x^3 + C$ $A = (\frac{1}{3}x^3 \ln 2x - \frac{1}{9}x^3)\Big\|_{\frac{1}{2}}^{1} = \frac{1}{3}\ln 2 - \frac{7}{72}$

50	$\frac{1}{16}x^3 = 2\sqrt{x} \Rightarrow \frac{1}{256}x^6 - 4x = 0$ $\Rightarrow x\left(\frac{1}{256}x^5 - 4\right) = 0 \Rightarrow x = 0,$ $x = \sqrt[5]{4(256)} = \sqrt[5]{2^{10}} = 4$ $A = \int_0^4 \left(2\sqrt{x} - \frac{1}{16}x^3\right)dx = \left(\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{64}x^4\right)\Big\|_0^4 = \frac{20}{3}$
51	$x^2 + 14 = x^4 + 2 \Rightarrow x^4 - x^2 - 12 = 0$ $\Rightarrow (x^2-4)(x^2+3) = 0 \Rightarrow x = \pm 2$ $\Rightarrow A(-2, f(-2)) = (-2, 18)$ $B(2, f(2)) = (2, 18)$
52	نلاحظ أن منحنيي f و g واقعان فوق المحور x ، وأن منحنى f فوق منحنى g في الفترة (-2,2) $\Rightarrow V = \pi\int_0^2 \left(f^2(x) - g^2(x)\right)dx$ $= \pi\int_0^2 ((x^2+14)^2 - (x^4+2)^2)\,dx$ $= \pi\int_0^2 (-x^8 - 3x^4 + 28x^2 + 192)dx$ $= \pi\left(-\frac{1}{9}x^9 - \frac{3}{5}x^5 + \frac{28}{3}x^3 + 192x\right)\Big\|_0^2 = \frac{17216\pi}{45}$
53	$V = \pi\int_1^2 (f(x))^2 dx = \pi\int_1^2 xe^{-x}dx$ $u = x \qquad dv = e^{-x}\,dx$ $du = dx \qquad v = -e^{-x}$ $\int xe^{-x}dx = -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + C$ $V = \pi\int_1^2 xe^{-x}dx = \pi\left((-xe^{-x} - e^{-x})\,\|_1^2\right) = \frac{2e-3}{e^2}\pi$
54	$\frac{dy}{\sqrt{y}} = \frac{dx}{x} \Rightarrow \int\frac{dy}{\sqrt{y}} = \int\frac{dx}{x} \Rightarrow 2\sqrt{y} = \ln\|x\| + C$

55	$\frac{dy}{\sec y} = xe^x dx \Rightarrow \int \cos y\ dy = \int xe^x dx$ $u = x \qquad dv = e^x\, dx$ $du = dx \qquad v = e^x$ $\Rightarrow \int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C$ $\Rightarrow \int \cos y\ dy = \int xe^x dx$ $\Rightarrow \sin y = xe^x - e^x + C$
56	$3y^2 dy = 8x dx$ $\int 3y^2 dy = \int 8x dx \Rightarrow y^3 = 4x^2 + C$
57	$x dy = \sqrt{y}(3x+4) dx$ $\frac{dy}{\sqrt{y}} = \frac{3x+4}{x} dx \Rightarrow \int y^{-\frac{1}{2}} dy = \int \left(3 + \frac{4}{x}\right) dx$ $\Rightarrow 2\sqrt{y} = 3x + 4\ln\lvert x\rvert + C$
58	$\frac{dy}{dx} = 8 - 4y = 4(2-y)$ $\frac{dy}{2-y} = 4dx \Rightarrow \int \frac{dy}{2-y} = \int 4dx$ الحل العام : $-\ln\lvert 4-y\rvert = 4x + C$ لإيجاد الحل الخاص نعوض $x = 0, y = 3$ في الحل العام: $-\ln 1 = 0 + C \Rightarrow C = 0$ الحل الخاص: $-\ln\lvert 4-y\rvert = 4x$

59	$\frac{dy}{5e^y} = \frac{dx}{(2x+1)(x-2)}$ $\frac{1}{(2x+1)(x-2)} = \frac{A}{2x+1} + \frac{B}{x-2}$ $\Rightarrow A(x-2) + B(2x+1) = 1$ $x = 2 \Rightarrow B = \frac{1}{5}$ $x = -\frac{1}{2} \Rightarrow A = -\frac{2}{5}$ $\int \frac{dy}{5e^y} = \int \left(\frac{-\frac{2}{5}}{2x+1} + \frac{\frac{1}{5}}{x-2} \right) dx$ $\frac{-e^{-y}}{5} = -\frac{1}{5}\ln\|2x+1\| + \frac{1}{5}\ln\|x-2\| + C$ الحل العام لإيجاد الحل الخاص نعوض $x = -3, y = 0$ في الحل العام: $\frac{-1}{5} = -\frac{1}{5}\ln 5 + \frac{1}{5}\ln 5 + C \Rightarrow C = \frac{-1}{5}$ $\frac{-e^{-y}}{5} = -\frac{1}{5}\ln\|2x+1\| + \frac{1}{5}\ln\|x-2\| - \frac{1}{5}$ الحل الخاص: $\Rightarrow \frac{1-e^{-y}}{5} = \frac{1}{5}\ln\left\|\frac{x-2}{2x+1}\right\| \Rightarrow 1 - e^{-y} = \ln\left\|\frac{x-2}{2x+1}\right\|$
60	$\frac{dx}{x} = 0.2\,dt \Rightarrow \int \frac{dx}{x} = \int 0.2\,dt$ $\Rightarrow \ln\|x\| = 0.2t + C \Rightarrow x = e^{0.2t+C} = e^C(e^{0.2t}) = Ke^{0.2t}$ حيث K ثابت يساوي e^C، وبملاحظة أن عدد الأسماك x أكبر من صفر (فيكون $\|x\| = x$) $x(0) = 300 \Rightarrow 300 = Ke^{0.2(0)} \Rightarrow K = 300$ الحل الخاص: $x(t) = 300e^{0.2t}$
61	$x(5) = 300\,e^{0.2(5)} = 300e \approx 815$ إذن، عدد الأسماك في البحيرة بعد خمس سنوات هو 815 سمكة تقريبًا.

62	$p(x) = \int \frac{-300x}{(9+x^2)^{\frac{3}{2}}}\, dx$ $u = 9 + x^2 \quad \Rightarrow dx = \frac{du}{2x}$ $p(u) = \int \frac{-300x}{u^{\frac{3}{2}}} \frac{du}{2x} = \int -150u^{\frac{-3}{2}}\, du = \frac{300}{\sqrt{u}} + C$ $\Rightarrow p(x) = \frac{300}{\sqrt{9+x^2}} + C$ $p(4) = \frac{300}{5} + C \Rightarrow 75 = 60 + C \Rightarrow C = 15$ $\Rightarrow p(x) = 15 + \frac{300}{\sqrt{9+x^2}}$