<table>
<tr><td colspan="2">مسألة اليوم صفحة 60</td></tr>
<tr><td></td><td>

$S(t)=\int 350\ln(t+1)\,dt$

$u=\ln(t+1) \qquad dv=350\,dt$

$du=\frac{1}{t+1}dt \qquad v=350\,t$

$\int udv=uv-\int vdu$

$\int 350\ln(1+t)dt=350t\ln(t+1)-\int\frac{350t}{t+1}dt$

$=350t\ln(t+1)-\int(350-\frac{350}{t+1})dt$

$=350t\ln(t+1)-350t+350\ln(t+1)+C$

$S(t)=0-0+0+C=0 \quad \Rightarrow C=0$

$S(t)=350t\ln(t+1)-350t+350\ln(t+1)$

</td></tr>
<tr><td colspan="2">أتحقق من فهمي صفحة 63</td></tr>
<tr><td>a</td><td>

$u=x \qquad dv=\sin x\ dx$

$du=dx \qquad v=-\cos x$

$\int x\sin x dx \ =-x\cos x-\int -\cos x\ dx$

$=-x\cos x+\sin x+C$

</td></tr>
<tr><td>b</td><td>

$u=\ln x \qquad dv=x^2\,dx$

$du=\frac{1}{x}\,dx \qquad v=\frac{1}{3}x^3$

$\int x^2\ln x\,dx\ =\frac{1}{3}x^3\ln x-\int\frac{1}{3}x^2\,dx$

$=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+C$

</td></tr>
</table>

	ملاحظة: يمكن حل هذه المسألة بطريقة التعويض $u = \sqrt{7-3x}$ أو $u = 7 - 3x$ وتاليًا حلّها بالأجزاء: $u = x \qquad dv = (7-3x)^{\frac{1}{2}}\,dx$ $du = dx \qquad v = -\frac{2}{9}(7-3x)^{\frac{3}{2}}$ $\int x\sqrt{7-3x}\,dx = -\frac{2}{9}x(7-3x)^{\frac{3}{2}} - \int -\frac{2}{9}(7-3x)^{\frac{3}{2}}\,dx$ $= -\frac{2}{9}x(7-3x)^{\frac{3}{2}} - \frac{4}{135}(7-3x)^{\frac{5}{2}} + C$
c	
d	$u = 3x \qquad dv = e^{4x}\,dx$ $du = 3dx \qquad v = \frac{1}{4}e^{4x}$ $\int 3xe^{4x}\,dx = \frac{3}{4}xe^{4x} - \int \frac{3}{4}e^{4x}\,dx$ $= \frac{3}{4}xe^{4x} - \frac{3}{16}e^{4x} + C$
أتحقق من فهمي صفحة 64	

a	$u = x^2 \qquad dv = \sin x\ dx$ $du = 2x\,dx \qquad v = -\cos x$ $\int x^2 \sin x\ dx = -x^2 \cos x - \int -2x \cos x\ dx$ $\int x^2 \sin x\ dx = -x^2 \cos x + \int 2x \cos x\ dx$ $u = 2x \qquad dv = \cos x\ dx$ $du = 2\,dx \qquad v = \sin x$ $\int x^2 \sin x\ dx = -x^2 \cos x + 2x \sin x - \int 2 \sin x\ dx$ $= -x^2 \cos x + 2x \sin x + 2 \cos x + C$
b	$u = x^3 \qquad dv = e^{4x}\,dx$ $du = 3x^2 dx \qquad v = \frac{1}{4}e^{4x}$ $\int x^3 e^{4x}\ dx = \frac{1}{4}x^3 e^{4x} - \int \frac{3}{4}x^2 e^{4x}\ dx$ $u = \frac{3}{4}x^2 \qquad dv = e^{4x}\,dx$ $du = \frac{3}{2}x\,dx \qquad v = \frac{1}{4}e^{4x}$ $\int x^3 e^{4x}\ dx = \frac{1}{4}x^3 e^{4x} - \frac{3}{16}x^2 e^{4x} + \int \frac{3}{8}x e^{4x}\ dx$ $u = \frac{3}{8}x \qquad dv = e^{4x}\ dx$ $du = \frac{3}{8}\ dx \qquad v = \frac{1}{4}e^{4x}$ $\int x^3 e^{4x}\ dx = \frac{1}{4}x^3 e^{4x} - \frac{3}{16}x^2 e^{4x} + \frac{3}{32}x e^{4x} - \int \frac{3}{32} e^{4x}\ dx$ $= \frac{1}{4}x^3 e^{4x} - \frac{3}{16}x^2 e^{4x} + \frac{3}{32}x e^{4x} - \frac{3}{128}e^{4x} + C$
أتحقق من فهمي صفحة 66	

a	$\int \frac{\sin x}{e^x} dx = \int \sin x\, e^{-x}\, dx$ $u = \sin x \qquad dv = e^{-x}\, dx$ $du = \cos x\, dx \qquad v = -e^{-x}$ $\int \sin x\, e^{-x}\, dx = -\sin x\, e^{-x} - \int -e^{-x} \cos x\, dx$ $\int \sin x\, e^{-x}\, dx = -\sin x\, e^{-x} + \int e^{-x} \cos x\, dx$ $u = \cos x \qquad dv = e^{-x}\, dx$ $du = -\sin x\, dx \qquad v = -e^{-x}$ $\int \sin x\, e^{-x}\, dx = -\sin x\, e^{-x} + e^{-x} \cos x - \int e^{-x} \sin x\, dx$ $\Rightarrow 2\int \sin x\, e^{-x}\, dx = e^{-x}(-\sin x + \cos x) + C$ $\int \sin x\, e^{-x}\, dx = \frac{1}{2} e^{-x}(-\sin x + \cos x) + C$
b	$u = \sec x \qquad dv = \sec^2 x\, dx$ $du = \sec x \tan x\, dx \qquad v = \tan x$ $\int \sec^3 x\, dx = \sec x \tan x - \int \sec x \tan^2 x\, dx$ $= \sec x \tan x - \int \sec x(\sec^2 x - 1)\, dx$ $= \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$ $2\int \sec^3 x\, dx = \sec x \tan x + \int \sec x\, dx$ $= \sec x \tan x + \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} dx$ $= \sec x \tan x + \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$ $= \sec x \tan x + \ln\lvert\sec x + \tan x\rvert$ $\int \sec^3 x\, dx = \frac{1}{2}(\sec x \tan x + \ln\lvert\sec x + \tan x\rvert) + C$

أتحقق من فهمي صفحة 67

a

افرض أن: $g(x) = \cos 4x$ ، $f(x) = x^4$ ، استخدم طريقة الجدول للتكامل بالأجزاء:

$f(x)$ ومشتقاته المتكررة	الإشارة	$g(x)$ وتكاملاته المتكررة
x^4	+	$\cos 4x$
$4x^3$	-	$\frac{1}{4}\sin 4x$
$12x^2$	+	$-\frac{1}{16}\cos 4x$
$24x$	-	$-\frac{1}{64}\sin 4x$
24	+	$\frac{1}{256}\cos 4x$
0		$\frac{1}{1024}\sin 4x$

$$\int x^4 \cos 4x\, dx =$$

$$\frac{1}{4}x^4 \sin 4x + \frac{1}{4}x^3 \cos 4x - \frac{3}{16}x^2 \sin 4x - \frac{3}{32}x \cos 4x + \frac{3}{128}\sin 4x + C$$

b

افرض أن: $g(x) = e^x$ ، $f(x) = x^5$ ، استخدم طريقة الجدول للتكامل بالأجزاء:

$f(x)$ ومشتقاته المتكررة	الإشارة	$g(x)$ وتكاملاته المتكررة
x^5	+	e^x
$5x^4$	-	e^x
$20x^3$	+	e^x
$60x^2$	-	e^x
$120x$	+	e^x
120	-	e^x
0		e^x

$$\int x^5 e^x\, dx = e^x\left(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120\right) + C$$

أتحقق من فهمي صفحة 69	
	$C(x)=\int(0.1x+1)e^{0.03x}\,dx$ $u=0.1x+1 \qquad dv=e^{0.03x}\,dx$ $du=0.1dx \qquad v=\frac{1}{0.03}e^{0.03x}$ $\int(0.1x+1)e^{0.03x}\,dx=(0.1x+1)\left(\frac{1}{0.03}e^{0.03x}\right)-\int\frac{0.1}{0.03}e^{0.03x}\,dx$ $=\frac{10}{3}(x+10)e^{0.03x}-\frac{1000}{9}e^{0.03x}+C$ $C(10)=\frac{200}{3}e^{0.3}-\frac{1000}{9}e^{0.3}+C=200 \Rightarrow C\approx 260$ $\Rightarrow C(x)=\frac{10}{3}e^{0.03x}\left(x-\frac{70}{3}\right)+260$
أتحقق من فهمي صفحة 70	
a	$u=\ln x \qquad dv=x^{-2}\,dx$ $du=\frac{1}{x}\,dx \qquad v=-\frac{1}{x}$ $\int_1^e\frac{\ln x}{x^2}\,dx=-\frac{\ln x}{x}\Big\|_1^e+\int_1^e x^{-2}\,dx$ $=-\frac{\ln x}{x}\Big\|_1^e+(-\frac{1}{x})\Big\|_1^e$ $=-\frac{1}{e}-\frac{1}{e}+1=1-\frac{2}{e}$
b	$u=x \qquad dv=e^{-2x}\,dx$ $du=dx \qquad v=-\frac{1}{2}e^{-2x}$ $\int_0^1 xe^{-2x}\,dx=-\frac{1}{2}xe^{-2x}\Big\|_0^1+\int_0^1\frac{1}{2}e^{-2x}\,dx$ $=-\frac{1}{2}xe^{-2x}\Big\|_0^1+-\frac{1}{4}e^{-2x}\Big\|_0^1$ $=-\frac{e^{-2}}{2}-\frac{e^{-2}}{4}+\frac{1}{4}=\frac{1}{4}-\frac{3}{4e^2}$
أتحقق من فهمي صفحة 71	

$$\int (x^3+x^5)\sin x^2\,dx = \int x^3 \sin x^2\,dx + \int x^5 \sin x^2\,dx$$

نجد كل تكامل على حدة. فنجد التكامل الأيسر كما يأتي:

$$y = x^2 \quad \Rightarrow \frac{dy}{dx} = 2x \Rightarrow dx = \frac{dy}{2x}$$

$$\int x^3 \sin x^2\,dx = \int x^3 \sin y\,\frac{dy}{2x} = \frac{1}{2}\int x^2 \sin y\,dy$$

$$= \frac{1}{2}\int y\sin y\,dy$$

$u = y \qquad dv = \sin y$

$du = dy \qquad v = -\cos y$

$$\int y\sin y\,dy = -y\cos y - \int -\cos y\,dy$$

$$= -y\cos y + \sin y$$

$$\int x^3 \sin x^2\,dx == -\frac{1}{2}x^2\cos x^2 + \frac{1}{2}\sin x^2 + C$$

a

ونجد التكامل الأيمن كما يأتي:

$$\int x^5 \sin x^2\,dx = \int x^5 \sin y\,\frac{dy}{2x} = \frac{1}{2}\int x^4 \sin y\,dy$$

$$= \frac{1}{2}\int y^2 \sin y\,dy$$

$u = y^2 \qquad dv = \sin y$

$du = 2y dy \qquad v = -\cos y$

$$\int y^2 \sin y\,dy = -y^2\cos y - \int -2y\cos y\,dy$$

$$= -y^2\cos y + 2y\sin y - 2\int \sin y\,dy$$

$$= -y^2\cos y + 2y\sin y + 2\cos y$$

$$\int x^5 \sin x^2\,dx = \frac{-1}{2}x^4\cos x^2 + x^2\sin x^2 + \cos x^2 + C$$

$$\int (x^3+x^5)\sin x^2\,dx = -\frac{1}{2}x^2\cos x^2 + \frac{1}{2}\sin x^2 - \frac{1}{2}x^4\cos x^2 + x^2\sin x^2 + \cos x^2 + C$$

b	$y = x^2 \Rightarrow \frac{dy}{dx} = 2x \Rightarrow dx = \frac{dy}{2x}$ $\int x^5 e^{x^2}\, dx = \int x^5 e^y \frac{dy}{2x} = \int \frac{1}{2} x^4 e^y\, dy = \frac{1}{2}\int y^2 e^y\, dy$ $u = y^2 \qquad dv = e^y\, dy$ $du = 2y dy \qquad v = e^y$ $\int y^2 e^y\, dy = y^2 e^y - \int 2y e^y\, dy$ $= y^2 e^y - 2ye^y + \int 2e^y\, dy$ $= y^2 e^y - 2ye^y + 2e^y = (y^2 - 2y + 2)e^y$ $\int x^5 e^{x^2}\, dx = (\frac{1}{2} x^4 - x^2 + 1)e^{x^2} + C$
	أتدرب وأحل المسائل صفحة
1	$u = x + 1 \qquad dv = \cos x\, dx$ $du = dx \qquad v = \sin x$ $\int (x+1)\cos x\, dx = (x+1)\sin x - \int \sin x\, dx$ $= (x+1)\sin x + \cos x + C$
2	$u = x \qquad dv = e^{\frac{1}{2}x}\, dx$ $du = dx \qquad v = 2e^{\frac{1}{2}x}$ $\int xe^{\frac{1}{2}x}\, dx = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\, dx$ $= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} + C$

3	$u = 2x^2 - 1 \qquad dv = e^{-x}\,dx$ $du = 4x\,dx \qquad v = -e^{-x}$ $\int (2x^2-1)e^{-x}\,dx = -(2x^2-1)e^{-x} + \int 4xe^{-x}\,dx$ بالأجزاء مرة أخرى: $u = 4x \qquad dv = e^{-x}\,dx$ $du = 4\,dx \qquad v = -e^{-x}$ $\int (2x^2-1)e^{-x}\,dx = -(2x^2-1)e^{-x} - 4xe^{-x} + \int 4e^{-x}\,dx$ $= -(2x^2-1)e^{-x} - 4xe^{-x} - 4e^{-x} + C$ $= -e^{-x}(2x^2+4x+3) + C$
4	$\int \ln\sqrt{x}\,dx = \int \frac{1}{2}\ln x\,dx$ $u = \ln x \qquad dv = \frac{1}{2}\,dx$ $du = \frac{1}{x}\,dx \qquad v = \frac{1}{2}x$ $\int \frac{1}{2}\ln x\,dx = \frac{1}{2}x\ln x - \int \frac{1}{2}\,dx$ $= \frac{1}{2}x\ln x - \frac{1}{2}x + C$
5	$\int x\sin x\cos x\,dx = \int \frac{1}{2}x\sin 2x\,dx$ $u = \frac{1}{2}x \qquad dv = \sin 2x\,dx$ $du = \frac{1}{2}\,dx \qquad v = -\frac{1}{2}\cos 2x$ $\int x\sin x\cos x\,dx = -\frac{1}{4}x\cos 2x + \int \frac{1}{4}\cos 2x\,dx$ $= -\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$

6	$u = x \qquad dv = \sec x \tan x \; dx$ $du = dx \qquad v = \sec x$ $\int x \sec x \tan x \; dx = x \sec x - \int \sec x \; dx$ $= x \sec x - \int \sec x \times \frac{\sec x + \tan x}{\sec x + \tan x} \; dx$ $= x \sec x - \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \; dx$ $= x \sec x - \ln\lvert\sec x + \tan x\rvert + C$
7	$\int \frac{x}{\sin^2 x} dx = \int x\csc^2 x \; dx$ $u = x \qquad dv = \csc^2 x \; dx$ $du = dx \qquad v = -\cot x$ $\int x\csc^2 x \; dx = -x \cot x + \int \cot x \; dx$ $= -x \cot x + \int \frac{\cos x}{\sin x} \; dx$ $= -x \cot x + \ln\lvert\sin x\rvert + C$
8	$u = \ln x \qquad dv = x^{-3} \; dx$ $du = \frac{1}{x} \; dx \qquad v = -\frac{1}{2} x^{-2}$ $\int x^{-3} \ln x \; dx = -\frac{1}{2} x^{-2} \ln x - \int -\frac{1}{2} x^{-2} \frac{1}{x} \; dx$ $= -\frac{1}{2} x^{-2} \ln x + \int \frac{1}{2} x^{-3} \; dx$ $= -\frac{1}{2} x^{-2} \ln x - \frac{1}{4} x^{-2} + C$ $= -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + C$

9	$u = 2x^2 \qquad dv = \sec^2 x \tan x \, dx$ $du = 4x\,dx \qquad v = \frac{1}{2}\tan^2 x$ ملاحظة: لإيجاد v استخدمنا طريقة التعويض، حيث: $y = \tan x , dx = \frac{dy}{\sec^2 x}$ ومنه: $v = \int \sec^2 x \tan x\,dx = \int \sec^2 x \, y \frac{dy}{\sec^2 x} = \int y dy = \frac{1}{2}y^2 = \frac{1}{2}\tan^2 x$ $\int 2x^2 \sec^2 x \tan x\,dx = 2x^2\left(\frac{1}{2}\tan^2 x\right) - \int 2x\tan^2 x\,dx$ بالأجزاء مرة أخرى: $u = 2x \qquad dv = \tan^2 x\,dx = (\sec^2 x - 1)dx$ $du = 2\,dx \qquad v = \tan x - x$ $\int 2x^2 \sec^2 x \tan x\,dx$ $= x^2\tan^2 x - \left(2x(\tan x - x) - \int 2\,(\tan x - x)dx\right)$ $= x^2\tan^2 x - 2x\tan x + 2x^2 + 2\int \left(\frac{\sin x}{\cos x} - x\right)dx$ $= x^2\tan^2 x - 2x\tan x + 2x^2 - 2\ln\lvert\cos x\rvert - x^2 + C$ $= x^2\tan^2 x - 2x\tan x + x^2 - 2\ln\lvert\cos x\rvert + C$
10	هذه المسألة يمكن حلها بالتعويض ($u = 8 - x$ أو $u = \sqrt{8-x}$) وحلها بالأجزاء كالآتي: $u = x - 2 \qquad dv = (8-x)^{\frac{1}{2}}\,dx$ $du = dx \qquad v = -\frac{2}{3}(8-x)^{\frac{3}{2}}$ $\int (x-2)\sqrt{8-x}dx = (x-2) \times -\frac{2}{3}(8-x)^{\frac{3}{2}} - \int -\frac{2}{3}(8-x)^{\frac{3}{2}}dx$ $= -\frac{2}{3}(x-2)(8-x)^{\frac{3}{2}} - \frac{4}{15}(8-x)^{\frac{5}{2}} + C$

11

بالأجزاء 3 مرات، نستخدم طريقة الجدول:

$f(x)$ ومشتقاته المتكررة	الإشارة	$g(x)$ وتكاملاته المتكررة
x^3		$\cos 2x$
$3x^2$	+	$\frac{1}{2}\sin 2x$
$6x$	-	$-\frac{1}{4}\cos 2x$
6	+	$-\frac{1}{8}\sin 2x$
0	-	$\frac{1}{16}\cos 2x$

$$\int x^3 \cos 2x \, dx = \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x - \frac{3}{8}\cos 2x + C$$

12

$$\int \frac{x}{6^x}dx = \int x6^{-x}dx$$

$u = x \qquad dv = 6^{-x}\,dx$

$du = dx \qquad v = -\frac{6^{-x}}{\ln 6}$

$$\int x6^{-x}dx = -x\frac{6^{-x}}{\ln 6} + \int \frac{6^{-x}}{\ln 6}dx$$

$$= -x\frac{6^{-x}}{\ln 6} - \frac{6^{-x}}{(\ln 6)^2} + C$$

13	$u = e^{-x} \qquad dv = \sin 2x \; dx$ $du = -e^{-x}\,dx \qquad v = \frac{-1}{2}\cos 2x$ $\int e^{-x}\sin 2x\,dx = -\frac{1}{2}e^{-x}\cos 2x - \int \frac{1}{2}e^{-x}\cos 2x\,dx$ بالأجزاء مرة أخرى: $u = \frac{1}{2}e^{-x} \qquad dv = \cos 2x \; dx$ $du = -\frac{1}{2}e^{-x}\,dx \qquad v = \frac{1}{2}\sin 2x$ $\int e^{-x}\sin 2x\,dx = -\frac{1}{2}e^{-x}\cos 2x - \frac{1}{4}e^{-x}\sin 2x - \frac{1}{4}\int e^{-x}\sin 2x\,dx$ $\int e^{-x}\sin 2x\,dx + \frac{1}{4}\int e^{-x}\sin 2x\,dx = -\frac{1}{4}e^{-x}(\sin 2x + 2\cos 2x) + C$ $\frac{5}{4}\int e^{-x}\sin 2x\,dx = -\frac{1}{4}e^{-x}(\sin 2x + 2\cos 2x) + C$ $\int e^{-x}\sin 2x\,dx = -\frac{1}{5}e^{-x}(\sin 2x + 2\cos 2x) + C$
14	$u = \ln \sin x \qquad dv = \cos x \; dx$ $du = \frac{\cos x}{\sin x}\,dx \qquad v = \sin x$ $\int \cos x \ln \sin x\,dx = \sin x \ln \sin x - \int \cos x\,dx$ $= \sin x \ln \sin x - \sin x + C$
15	$u = \ln(1+e^{x}) \qquad dv = e^{x}\,dx$ $du = \frac{e^{x}}{1+e^{x}}\,dx \qquad v = e^{x}$ $\int e^{x}\ln(1+e^{x})\,dx = e^{x}\ln(1+e^{x}) - \int \frac{e^{2x}}{1+e^{x}}\,dx$ $= e^{x}\ln(1+e^{x}) - \int \left(e^{x} + \frac{-1}{1+e^{x}}\right)dx$ $= e^{x}\ln(1+e^{x}) - \int \left(e^{x} + \frac{-e^{-x}}{e^{-x}+1}\right)dx$ $= e^{x}\ln(1+e^{x}) - e^{x} - \ln(1+e^{-x}) + C$

16	وجدنا في المثال 3 أنّ: $\int e^x \cos x\, dx = \frac{1}{2}e^x(\sin x + \cos x) + C$ $\Rightarrow \int_0^{\frac{\pi}{2}} e^x \cos x\, dx = \frac{1}{2}e^x(\sin x + \cos x)\Big\|_0^{\frac{\pi}{2}}$ $= \frac{1}{2}e^{\frac{\pi}{2}} - \frac{1}{2}e^0 = \frac{1}{2}e^{\frac{\pi}{2}} - \frac{1}{2}$
17	$\int_1^e \ln x^2\, dx = \int_1^e 2\ln x\, dx$ $u = 2\ln x \qquad dv = dx$ $du = \frac{2}{x}\, dx \qquad v = x$ $\int_1^e 2\ln x\, dx = 2x\ln x\|_1^e - \int_1^e 2dx$ $= 2x\ln x\|_1^e - 2x\|_1^e$ $= 2e\ln e - 2\ln 1 - 2e + 2 = 2e - 0 - 2e + 2 = 2$
18	$\int_1^2 \ln(xe^x)\, dx = \int_1^2 (\ln x + \ln e^x) dx$ $= \int_1^2 (\ln x + x) dx = \int_1^2 \ln x\, dx + \int_1^2 x dx$ نجد $\int_1^2 \ln x\, dx$ بطريقة الأجزاء: $u = \ln x \qquad dv = dx$ $du = \frac{1}{x}\, dx \qquad v = x$ $\int_1^2 \ln x\, dx = x\ln x\|_1^2 - \int_1^2 dx = x\ln x\|_1^2 - x\|_1^2 = 2\ln 2 - \ln 1 - 2 + 1$ $= 2\ln 2 - 1$ $\int_1^2 x dx = \frac{1}{2}x^2\Big\|_1^2 = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$ $\Rightarrow \int_1^2 \ln(xe^x)\, dx = 2\ln 2 - 1 + \frac{3}{2} = 2\ln 2 + \frac{1}{2}$

19	$u = x \qquad dv = \sec^2 3x\, dx$ $du = dx \qquad v = \frac{1}{3}\tan 3x$ $\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} x\sec^2 3x\, dx = \frac{1}{3}x\tan 3x\Big\|_{\frac{\pi}{12}}^{\frac{\pi}{9}} - \int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \frac{1}{3}\tan 3x\, dx$ $= \frac{1}{3}x\tan 3x\Big\|_{\frac{\pi}{12}}^{\frac{\pi}{9}} - \int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \frac{1}{3}\frac{\sin 3x}{\cos 3x}\, dx$ $= \frac{1}{3}x\tan 3x\Big\|_{\frac{\pi}{12}}^{\frac{\pi}{9}} + \frac{1}{9}\ln\cos 3x\Big\|_{\frac{\pi}{12}}^{\frac{\pi}{9}}$ $= \frac{\pi}{27}\tan\frac{\pi}{3} - \frac{\pi}{36}\tan\frac{\pi}{4} + \frac{1}{9}\ln\cos\frac{\pi}{3} - \frac{1}{9}\ln\cos\frac{\pi}{4}$ $= \frac{\pi\sqrt{3}}{27} - \frac{\pi}{36} + \frac{1}{9}\ln\frac{1}{2} - \frac{1}{9}\ln\frac{1}{\sqrt{2}}$
20	$u = \ln x \qquad dv = x^4\, dx$ $du = \frac{dx}{x} \qquad v = \frac{1}{5}x^5$ $\int_1^e x^4 \ln x\, dx = \frac{1}{5}x^5\ln x\Big\|_1^e - \int_1^e \frac{1}{5}x^4\, dx$ $= \frac{1}{5}x^5\ln x\Big\|_1^e - \frac{1}{25}x^5\Big\|_1^e$ $= \frac{1}{5}e^5 - 0 - \frac{1}{25}e^5 + \frac{1}{25} = \frac{4e^5+1}{25}$

21	نجد $\int x^2 \sin x\, dx$ باستخدام طريقة الجدول: $f(x)$ ومشتقاته المتكررة — $g(x)$ وتكاملاته المتكررة x^2 → (+) → $-\cos x$ (مع $\sin x$) $2x$ → (-) → $-\sin x$ 2 → (+) → $\cos x$ 0 $\int x^2 \sin x\, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$ $\Rightarrow \int_0^{\frac{\pi}{2}} x^2 \sin x\, dx = -x^2 \cos x + 2x \sin x + 2\cos x \Big\|_0^{\frac{\pi}{2}}$ $= \pi - 2$
22	$u = x \qquad dv = (e^{-2x} + e^{-x})\, dx$ $du = dx \qquad v = -\frac{1}{2}e^{-2x} - e^{-x}$ $\int_0^1 x(e^{-2x} + e^{-x})\, dx = -\frac{1}{2}xe^{-2x} - xe^{-x}\Big\|_0^1 - \int_0^1 \left(-\frac{1}{2}e^{-2x} - e^{-x}\right) dx$ $= -\frac{1}{2}xe^{-2x} - xe^{-x}\Big\|_0^1 - \left(\frac{1}{4}e^{-2x} + e^{-x}\right)\Big\|_0^1$ $= -\frac{1}{2}e^{-2} - e^{-1} - \frac{1}{4}e^{-2} - e^{-1} + \frac{1}{4} + 1$ $= -\frac{3}{4}e^{-2} - 2e^{-1} + \frac{5}{4}$
23	$u = xe^x \qquad dv = (1+x)^{-2}\, dx$ $du = (xe^x + e^x)\, dx = e^x(x+1)\, dx \qquad v = -(1+x)^{-1}$ $\int_0^1 \frac{xe^x}{(1+x)^2}\, dx = -xe^x(1+x)^{-1}\Big\|_0^1 + \int_0^1 \frac{e^x(x+1)}{(1+x)}\, dx$ $= -\frac{xe^x}{1+x}\Big\|_0^1 + e^x\Big\|_0^1$ $= -\frac{e}{2} + e - 1 = \frac{1}{2}e - 1$

24	$u = x \qquad dv = 3^x\,dx$ $du = dx \qquad v = \dfrac{3^x}{\ln 3}$ $\int_0^1 x3^x\,dx = x\dfrac{3^x}{\ln 3}\Big\vert_0^1 - \int_0^1 \dfrac{3^x}{\ln 3}\,dx$ $= x\dfrac{3^x}{\ln 3}\Big\vert_0^1 - \dfrac{3^x}{(\ln 3)^2}\Big\vert_0^1$ $= \dfrac{3}{\ln 3} - \dfrac{3}{(\ln 3)^2} + \dfrac{1}{(\ln 3)^2} = \dfrac{3\ln 3 - 2}{(\ln 3)^2}$
25	$y = x^2 \Rightarrow dx = \dfrac{dy}{2x}$ $\int x^3e^{x^2}\,dx = \int x^3e^y\dfrac{dy}{2x} = \int \dfrac{1}{2}x^2e^y\,dy = \int \dfrac{1}{2}ye^y\,dy$ $u = \dfrac{1}{2}y \qquad dv = e^y\,dy$ $du = \dfrac{1}{2}\,dy \qquad v = e^y$ $\int \dfrac{1}{2}ye^y\,dy = \dfrac{1}{2}ye^y - \int \dfrac{1}{2}e^y\,dy$ $= \dfrac{1}{2}ye^y - \dfrac{1}{2}e^y + C$ $\int x^3e^{x^2}\,dx = \dfrac{1}{2}x^2e^{x^2} - \dfrac{1}{2}e^{x^2} + C$
26	$y = \ln x \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x} \Rightarrow dx = x\,dy \quad , \quad x = e^y$ $\int \cos(\ln x)\,dx = \int x\cos y\,dy = \int e^y\cos y\,dy$ من المثال محلول الصفحات 55و56 في كتاب الطالب نجد أنّ: $\int e^y\cos y\,dy = \dfrac{1}{2}e^y(\sin y + \cos y) + C$ $\Rightarrow \int \cos(\ln x)\,dx = \dfrac{1}{2}e^{\ln x}(\sin\ln x + \cos\ln x) + C$ $= \dfrac{1}{2}x(\sin\ln x + \cos\ln x) + C$

27	$y = x^2 \Rightarrow dx = \dfrac{dy}{2x}$ $\int x^3 \sin x^2\, dx = \int x^3 \sin y \dfrac{dy}{2x} = \int \dfrac{1}{2} x^2 \sin y\, dy = \int \dfrac{1}{2} y \sin y\, dy$ $u = \dfrac{1}{2} y \qquad dv = \sin y\, dy$ $du = \dfrac{1}{2}\, dy \qquad v = -\cos y$ $\int \dfrac{1}{2} y \sin y\, dy = -\dfrac{1}{2} y \cos y + \int \dfrac{1}{2} \cos y\, dy$ $= -\dfrac{1}{2} y \cos y + \dfrac{1}{2} \sin y + C$ $\int x^3 \sin x^2\, dx = -\dfrac{1}{2} x^2 \cos x^2 + \dfrac{1}{2} \sin x^2 + C$
28	$y = \cos x \Rightarrow dx = \dfrac{dy}{-\sin x}$ $\int e^{\cos x} \sin 2x\, dx = \int e^y (2 \sin x \cos x) \dfrac{dy}{-\sin x} = \int -2ye^y dy$ $u = -2y \qquad dv = e^y dy$ $du = -2\, dy \qquad v = e^y$ $\int -2ye^y dy = -2ye^y + \int 2e^y dy$ $= -2ye^y + 2e^y + C$ $\Rightarrow \int e^{\cos x} \sin 2x\, dx = -2 \cos x\, e^{\cos x} + 2e^{\cos x} + C$

29	$y=\sqrt{x} \Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{x}}=\frac{1}{2y} \Rightarrow dx=2y\,dy$ $\int \sin\sqrt{x}\,dx=\int 2y\sin y\,dy$ $u=2y \qquad dv=\sin y\,dy$ $du=2\,dy \qquad v=-\cos y$ $\int 2y\sin y\,dy=-2y\cos y+\int 2\cos y\,dy$ $=-2y\cos y+2\sin y+C$ $\Rightarrow \int \sin\sqrt{x}\,dx=-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}+C$
30	$y=x^2 \Rightarrow \frac{dy}{dx}=2x \Rightarrow dx=\frac{dy}{2x}$ $\int \frac{x^3e^{x^2}}{(x^2+1)^2}dx=\int \frac{x^3e^y}{(y+1)^2}\frac{dy}{2x}=\int \frac{1}{2}x^2\frac{e^y}{(y+1)^2}dy=\int \frac{\frac{1}{2}ye^y}{(y+1)^2}dy$ $u=\frac{1}{2}ye^y \qquad dv=\frac{1}{(y+1)^2}dy$ $du=\frac{1}{2}(ye^y+e^y)\,dy=\frac{1}{2}e^y(y+1)\,dy \qquad v=\frac{-1}{y+1}$ $\int \frac{\frac{1}{2}ye^y}{(y+1)^2}dy=\frac{-ye^y}{2(y+1)}+\int \frac{1}{y+1}\times\frac{1}{2}e^y(y+1)dy$ $=\frac{-ye^y}{2(y+1)}+\frac{1}{2}\int e^y dy$ $=\frac{-ye^y}{2(y+1)}+\frac{1}{2}e^y+C$ $\int \frac{x^3e^{x^2}}{(x^2+1)^2}dx=\frac{-x^2e^{x^2}}{2(x^2+1)}+\frac{1}{2}e^{x^2}+C=\frac{e^{x^2}}{2(x^2+1)}+C$

31	الإحداثيان x للنقطتين A و B هما أصغر حلين موجبين للمعادلة: $f(x) = e^{-x}\sin 2x = 0$ $\Rightarrow \sin 2x = 0 \Rightarrow 2x = \pi, 2\pi, \ldots$ $\Rightarrow x = \frac{\pi}{2}, \pi, \ldots$ $\Rightarrow A\left(\frac{\pi}{2}, 0\right), B(\pi, 0)$

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$$A=\int_0^{\frac{\pi}{2}} e^{-x}\sin 2x\ dx+(-\int_{\frac{\pi}{2}}^{\pi} e^{-x}\sin 2x\ dx)$$

للتبسيط سنجد أولًا: $\int e^{-x}\sin 2x\ dx$ (التكامل غير المحدود)

$u=e^{-x}$ $\quad dv=\sin 2x\,dx$

$du=-e^{-x}dx$ $\quad v=-\frac{1}{2}\cos 2x$

$$\int e^{-x}\sin 2x\ dx=-\frac{1}{2}e^{-x}\cos 2x-\int \frac{1}{2}e^{-x}\cos 2x\ dx$$

بالأجزاء مرة أخرى:

$u=\frac{1}{2}e^{-x}$ $\quad dv=\cos 2x\,dx$

$du=-\frac{1}{2}e^{-x}dx$ $\quad v=\frac{1}{2}\sin 2x$

$$\int e^{-x}\sin 2x\ dx=-\frac{1}{2}e^{-x}\cos 2x-\frac{1}{4}e^{-x}\sin 2x-\frac{1}{4}\int e^{-x}\sin 2x\ dx$$

$$\int e^{-x}\sin 2x\ dx+\frac{1}{4}\int e^{-x}\sin 2x\ dx=-\frac{1}{2}e^{-x}\cos 2x-\frac{1}{4}e^{-x}\sin 2x$$

$$\frac{5}{4}\int e^{-x}\sin 2x\ dx=-\frac{1}{2}e^{-x}\cos 2x-\frac{1}{4}e^{-x}\sin 2x+C$$

$$\int e^{-x}\sin 2x\ dx=-\frac{1}{5}e^{-x}(2\cos 2x+\sin 2x)+C$$

$$\Rightarrow A=-\frac{1}{5}e^{-x}(2\cos 2x+\sin 2x)\Big|_0^{\frac{\pi}{2}}+\frac{1}{5}e^{-x}(2\cos 2x+\sin 2x)\Big|_{\frac{\pi}{2}}^{\pi}$$

$$=\frac{2}{5}e^{-\frac{\pi}{2}}+\frac{2}{5}+\frac{2}{5}e^{-\pi}+\frac{2}{5}e^{-\frac{\pi}{2}}$$

$$=\frac{2}{5}\left(1+e^{-\pi}+2e^{-\frac{\pi}{2}}\right)$$

33	$s(t) = \int te^{-\frac{t}{2}}dt$ $u = t \qquad dv = e^{-\frac{t}{2}}dt$ $du = dt \qquad v = -2e^{-\frac{t}{2}}$ $s(t) = -2te^{-\frac{t}{2}} - \int -2e^{-\frac{t}{2}}dt = -2te^{-\frac{t}{2}} - 4e^{-\frac{t}{2}} + C$ $s(0) = 0 - 4 + C$ $0 = 0 - 4 + C \Rightarrow C = 4$ $\Rightarrow s(t) = -2te^{-\frac{t}{2}} - 4e^{-\frac{t}{2}} + 4$
34	$f(x) = \int (x+2)\sin x \, dx$ $u = x + 2 \qquad dv = \sin x \, dx$ $du = dx \qquad v = -\cos x$ $f(x) = -(x+2)\cos x + \int \cos x \, dx$ $= -(x+2)\cos x + \sin x + C$ $f(0) = -2 + 0 + C$ $2 = -2 + 0 + C \Rightarrow C = 4$ $f(x) = -(x+2)\cos x + \sin x + 4$
35	$f(x) = \int 2xe^{-x} \, dx$ $u = 2x \qquad dv = e^{-x} \, dx$ $du = 2dx \qquad v = -e^{-x}$ $f(x) = -2xe^{-x} + \int 2e^{-x} \, dx$ $= -2xe^{-x} - 2e^{-x} + C$ $f(0) = 0 - 2 + C$ $3 = -2 + C \Rightarrow C = 5$ $f(x) = -2xe^{-x} - 2e^{-x} + 5$

36	$N(t)=\int (t+6)e^{-0.25t}dt$ $u=t+6 \qquad dv=e^{-0.25t}dt$ $du=dt \qquad v=-4e^{-0.25t}$ $N(t)=-4(t+6)e^{-0.25t}+\int 4e^{-0.25t}dt$ $=-4(t+6)e^{-0.25t}-16e^{-0.25t}+C$ $N(0)=-24-16+C$ $40=-24-16+C \Rightarrow C=80$ $\Rightarrow N(t)=-4(t+6)e^{-0.25t}-16e^{-0.25t}+80$
37	$u=\ln 2x \qquad dv=x^2dx$ $du=\frac{1}{x}dx \qquad v=\frac{1}{3}x^3$ $\int_{\frac{1}{2}}^{3} x^2\ln 2x\ dx=\frac{1}{3}x^3\ln 2x\Big\|_{\frac{1}{2}}^{3}-\int_{\frac{1}{2}}^{3}\frac{1}{3}x^2\ dx$ $=\frac{1}{3}x^3\ln 2x\Big\|_{\frac{1}{2}}^{3}-\frac{1}{9}x^3\Big\|_{\frac{1}{2}}^{3}$ $=9\ln 6-3+\frac{1}{72}=9\ln 6-\frac{215}{72}$

38	$u = x \qquad dv = \sin 5x \sin 3x\, dx = \frac{1}{2}(\cos 2x - \cos 8x)dx$ $du = dx \qquad v = \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x$ $\int_0^{\frac{\pi}{4}} x \sin 5x \sin 3x\; dx$ $= x\left(\frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x\right)\Big\|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}}\left(\frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x\right)dx$ $= x\left(\frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x\right)\Big\|_0^{\frac{\pi}{4}} - \left(-\frac{1}{8}\cos 2x + \frac{1}{128}\cos 8x\right)\Big\|_0^{\frac{\pi}{4}}$ $= \frac{\pi}{4}\left(\frac{1}{4}\right) + 0 - \frac{1}{128} - \frac{1}{8} + \frac{1}{128} = \frac{\pi - 2}{16}$
39	$u = x \qquad dv = e^{\frac{1}{2}x}dx$ $du = dx \qquad v = 2e^{\frac{1}{2}x}$ $\int_0^a xe^{\frac{1}{2}x}\, dx = 2xe^{\frac{1}{2}x}\Big\|_0^a - \int_0^a 2e^{\frac{1}{2}x}\, dx$ $= 2xe^{\frac{1}{2}x}\Big\|_0^a - 4e^{\frac{1}{2}x}\Big\|_0^a$ $= 2ae^{\frac{1}{2}a} - 4e^{\frac{1}{2}a} + 4$ $\Rightarrow 2ae^{\frac{1}{2}a} - 4e^{\frac{1}{2}a} + 4 = 6$ $2ae^{\frac{1}{2}a} = 4e^{\frac{1}{2}a} + 2$ بقسمة طرفي المعادلة على $2e^{\frac{1}{2}a}$ نحصل على: $a = 2 + e^{-\frac{1}{2}a}$ لذا فإن a يحقق المعادلة $x = 2 + e^{-\frac{x}{2}}$

الطريقة الأولى بالتعويض:

$$u = \ln x \;\Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du \quad , \quad x = e^u$$

$$\int (\ln x)^2 dx = \int u^2 x du = \int u^2 e^u du$$

بالأجزاء مرتين، نستخدم الجدول:

$f(u)$ ومشتقاته المتكررة		$g(u)$ وتكاملاته المتكررة
u^2	+	e^u
$2u$	-	e^u
2	+	e^u
0		e^u

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$$\int u^2 e^u du = e^u(u^2 - 2u + 2) + C$$

$$\int (\ln x)^2 dx = e^{\ln x}((\ln x)^2 - 2\ln x + 2) + C$$

$$= x((\ln x)^2 - 2\ln x + 2) + C$$

الطريقة الثانية: بالأجزاء مباشرة:

$$u = (\ln x)^2 \qquad dv = dx$$

$$du = \frac{2\ln x}{x} dx \qquad v = x$$

$$\int (\ln x)^2 dx = x(\ln x)^2 - \int 2\ln x\, dx$$

بالأجزاء مرة أخرى:

$$u = 2\ln x \qquad dv = dx$$

$$du = \frac{2}{x} dx \qquad v = x$$

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2x\ln x + \int 2dx$$

$$= x(\ln x)^2 - 2x\ln x + 2x + C$$

41	$A_1 = -\int_{-\frac{1}{2}}^{0} xe^{2x}\,dx$, $A_2 = \int_{0}^{\frac{1}{2}} xe^{2x}\,dx$ نجد التكامل غير المحدود $\int xe^{2x}\,dx$ بالأجزاء: $u = x \quad dv = e^{2x}dx$ $du = dx \quad v = \frac{1}{2}e^{2x}$ $\int xe^{2x}\,dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$ $= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$ $= \frac{1}{4}e^{2x}(2x-1) + C$ $\Rightarrow A(R_1) = -\frac{1}{4}e^{2x}(2x-1)\Big\vert_{-\frac{1}{2}}^{0} = \frac{1}{4} - \frac{1}{2e} = \frac{e-2}{4e}$ $A(R_2) = \frac{1}{4}e^{2x}(2x-1)\Big\vert_{0}^{\frac{1}{2}} = 0 + \frac{1}{4} = \frac{1}{4}$
42	$\frac{A(R_1)}{A(R_2)} = \frac{\frac{e-2}{4e}}{\frac{1}{4}} = \frac{e-2}{e}$ $A(R_1) : A(R_{12}) = (e-2) : e$
43	$u = \ln x \quad dv = x^n dx$ $du = \frac{1}{x}dx \quad v = \frac{1}{n+1}x^{n+1}$ $\int x^n \ln x\,dx = \frac{x^{n+1}\ln x}{n+1} - \int \frac{1}{n+1}x^n dx$ $= \frac{x^{n+1}\ln x}{n+1} - \frac{1}{(n+1)^2}x^{n+1} + C$ $= \frac{x^{n+1}}{(n+1)^2}(-1 + (n+1)\ln x) + C$

44	$u = x^n$ $dv = e^{ax}dx$ $du = nx^{n-1}dx$ $v = \frac{1}{a}e^{ax}$ $\int x^n e^{ax}dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}dx$