الدرس الثاني: التكامل بالتعويض

مسألة اليوم صفحة 28

$$G(t)=\int \frac{60000e^{-0.6t}}{(1+5e^{-0.6t})^2}dt$$

افرض أنّ: $u=1+5e^{-0.6t}$

$$\frac{du}{dt}=-3e^{-0.6t} \Rightarrow dt=\frac{du}{-3e^{-0.6t}}$$

$$G(t)=\int \frac{60000e^{-0.6t}}{u^2}\times\frac{du}{-3e^{-0.6t}}$$

$$=\int -20000u^{-2}du$$

$$=20000u^{-1}+C$$

$$G(t)=\frac{20000}{1+5e^{-0.6t}}+C$$

$$G(0)=\frac{20000}{1+5}+C$$

$$25000=\frac{10000}{3}+C \Rightarrow C=\frac{65000}{3}$$

$$G(t)=\frac{20000}{1+5e^{-0.6t}}+\frac{65000}{3}$$

$$G(20)=\frac{20000}{1+5e^{-12}}+\frac{65000}{3}\approx 41666\text{ kg}$$

أتحقق من فهمي صفحة 32

a	$u = x^3 - 5 \quad \Rightarrow \dfrac{du}{dx} = 3x^2 \Rightarrow dx = \dfrac{du}{3x^2}$ $\int 4x^2\sqrt{x^3 - 5}dx = \int 4x^2\sqrt{u} \times \dfrac{du}{3x^2}$ $= \int \dfrac{4}{3}u^{\frac{1}{2}}du$ $= \dfrac{8}{9}u^{\frac{3}{2}} + C$ $= \dfrac{8}{9}\sqrt{(x^3 - 5)^3} + C$
b	$u = \sqrt{x} \quad \Rightarrow \dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}} \quad \Rightarrow dx = 2\sqrt{x}du$ $\int \dfrac{1}{2\sqrt{x}}e^{\sqrt{x}}dx = \int \dfrac{1}{2\sqrt{x}}e^{u} \times 2\sqrt{x}du$ $= \int e^{u}du$ $= e^{u} + C$ $= e^{\sqrt{x}} + C$
c	$u = \ln x \quad \Rightarrow \dfrac{du}{dx} = \dfrac{1}{x} \quad \Rightarrow dx = xdu$ $\int \dfrac{(\ln x)^3}{x}dx = \int \dfrac{u^3}{x} \times xdu$ $= \int u^3du$ $= \dfrac{1}{4}u^4 + C$ $= \dfrac{1}{4}(\ln x)^4 + C$

d	$u = \ln x \quad \Rightarrow \frac{du}{dx} = \frac{1}{x} \quad \Rightarrow dx = x du$ $\int \frac{\cos(\ln x)}{x} dx = \int \frac{\cos u}{x} \times x du$ $= \int \cos u \, du$ $= \sin u + C$ $= \sin(\ln x) + C$
e	$u = \cos 5x \quad \Rightarrow \frac{du}{dx} = -5 \sin 5x \quad \Rightarrow dx = \frac{du}{-5 \sin 5x}$ $\int \cos^4 5x \, \sin 5x \, dx = \int u^4 \sin 5x \times \frac{du}{-5 \sin 5x}$ $= \int -\frac{1}{5} u^4 du$ $= -\frac{1}{25} u^5 + C$ $= -\frac{1}{25} \cos^5 5x + C$
f	$u = x^2 \quad \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ $\int x 2^{x^2} dx = \int x 2^u \times \frac{du}{2x}$ $= \int \frac{1}{2} 2^u du$ $= \frac{1}{2} \frac{2^u}{\ln 2} + C$ $= \frac{1}{\ln 2} 2^{x^2-1} + C$

أتحقق من فهمي صفحة 34

a	$u = 1 + 2x \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2} \quad , x = \frac{u-1}{2}$ $\int \frac{x}{\sqrt{1+2x}} dx = \int \frac{\frac{1}{2}(u-1)}{u^{\frac{1}{2}}} \times \frac{du}{2}$ $= \frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$ $= \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) + C$ $= \frac{1}{6}(1+2x)^{\frac{3}{2}} - \frac{1}{2}(1+2x)^{\frac{1}{2}} + C$ $= \frac{1}{6}\sqrt{(1+2x)^3} - \frac{1}{2}\sqrt{1+2x} + C$
b	$u = x^4 - 8 \Rightarrow \frac{du}{dx} = 4x^3 \Rightarrow dx = \frac{du}{4x^3} \quad , x^4 = u + 8$ $\int x^7(x^4-8)^3 dx = \int x^7 u^3 \times \frac{du}{4x^3}$ $= \frac{1}{4}\int x^4 u^3 du$ $= \frac{1}{4}\int (u+8)u^3 du$ $= \frac{1}{4}\int (u^4 + 8u^3) du$ $= \frac{1}{4}\left(\frac{1}{5}u^5 + 2u^4\right) + C$ $= \frac{1}{20}(x^4-8)^5 + \frac{1}{2}(x^4-8)^4 + C$

c	$u = 1 - e^x \Rightarrow \frac{du}{dx} = -e^x \Rightarrow dx = \frac{du}{-e^x} \quad , e^x = 1 - u$ $\int \frac{e^{3x}}{(1-e^x)^2} dx = \int \frac{e^{3x}}{u^2} \times \frac{du}{-e^x}$ $= \int -\frac{e^{2x}}{u^2} du$ $= \int \frac{-(1-u)^2}{u^2} du$ $= \int \frac{-1 + 2u - u^2}{u^2} du$ $= \int \left(-u^{-2} + \frac{2}{u} - 1\right) du$ $= (u^{-1} + 2\ln\lvert u\rvert - u) + C$ $= \frac{1}{1-e^x} + 2\ln\lvert 1 - e^x\rvert - 1 + e^x + C$
أتحقق من فهمي صفحة 35	
a	$u = \sqrt[3]{x} \Rightarrow \frac{du}{dx} = \frac{1}{3} x^{-\frac{2}{3}} \Rightarrow dx = 3x^{\frac{2}{3}} du \quad , \quad x = u^3$ $\int \frac{dx}{x + \sqrt[3]{x}} = \int \frac{3x^{\frac{2}{3}} du}{u^3 + u}$ $= \int \frac{3u^2}{u^3 + u} du$ $= \int \frac{3u}{u^2 + 1} du$ $= \frac{3}{2} \int \frac{2u}{u^2 + 1} du$ $= \frac{3}{2} \ln(u^2 + 1) + C$ $= \frac{3}{2} \ln\left(x^{\frac{2}{3}} + 1\right) + C$

b	$u = 1 - x \quad \Rightarrow \dfrac{du}{dx} = -1 \Rightarrow dx = -du \quad , \qquad x = 1 - u$ $\int x \sqrt[3]{(1-x)^2}\, dx = \int x \sqrt[3]{u^2} \times -du$ $= \int -(1-u)\sqrt[3]{u^2}\, du$ $= \int -(1-u) u^{\frac{2}{3}} du$ $= \int \left(-u^{\frac{2}{3}} + u^{\frac{5}{3}}\right) du$ $= -\frac{3}{5} u^{\frac{5}{3}} + \frac{3}{8} u^{\frac{8}{3}} + C$ $= -\frac{3}{5}(1-x)^{\frac{5}{3}} + \frac{3}{8}(1-x)^{\frac{8}{3}} + C$ $= -\frac{3}{5}\sqrt[3]{(1-x)^5} + \frac{3}{8}\sqrt[3]{(1-x)^8} + C$
أتحقق من فهمي صفحة 37	
	$p(x) = \int \dfrac{-135x}{\sqrt{9+x^2}} dx$ $u = 9 + x^2 \quad \Rightarrow \dfrac{du}{dx} = 2x \Rightarrow dx = \dfrac{du}{2x}$ $p(x) = \int \dfrac{-135x}{\sqrt{u}} \times \dfrac{du}{2x}$ $= \dfrac{-135}{2} \int u^{-\frac{1}{2}} du$ $= -135 u^{\frac{1}{2}} + C$ $p(x) = -135\sqrt{9+x^2} + C$ $p(4) = -135\sqrt{9+16} + C = -135(5) + C$ $30 = -675 + C \Rightarrow C = 705$ $p(x) = 705 - 135\sqrt{9+x^2}$
أتحقق من فهمي صفحة 39	

a	$\int \sin^3 x\, dx = \int \sin x \sin^2 x\, dx = \int \sin x\,(1-\cos^2 x)dx$ $u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$ $\int \sin^3 x\, dx = \int \sin x\,(1-u^2)\frac{du}{-\sin x}$ $= \int (u^2-1)du$ $= \frac{1}{3}u^3 - u + C$ $= \frac{1}{3}\cos^3 x - \cos x + C$
b	$u = \sin x \Rightarrow \frac{du}{dx} = \cos x \Rightarrow dx = \frac{du}{\cos x}$ $\int \cos^5 x \sin^2 x\, dx = \int \cos^5 x\, u^2 \frac{du}{\cos x}$ $= \int \cos^4 x\, u^2\, du$ $= \int (1-\sin^2 x)^2 u^2\, du$ $= \int (1-u^2)^2 u^2\, du$ $= \int (u^2 - 2u^4 + u^6)\, du$ $= \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$ $= \frac{1}{3}\sin^3 x - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C$

أتحقق من فهمي صفحة 41	
a	$\int \tan^4 x\ dx = \int \tan^2 x \tan^2 x\, dx$ $= \int \tan^2 x\,(\sec^2 x - 1) dx$ $= \int \tan^2 x \sec^2 x - \tan^2 x\, dx$ $= \int \tan^2 x \sec^2 x\, dx - \int \tan^2 x\, dx$ $= \int \tan^2 x \sec^2 x\, dx - \int (\sec^2 x - 1)\, dx$ $u = \tan x \Rightarrow \frac{du}{dx} = \sec^2 x \Rightarrow dx = \frac{du}{\sec^2 x}$ $\Rightarrow \int \tan^4 x\ dx = \int u^2 \sec^2 x \times \frac{du}{\sec^2 x} - \int (\sec^2 x - 1)\, dx$ $= \int u^2 du - \int (\sec^2 x - 1)\, dx$ $= \frac{1}{3} u^3 - \tan x + x + C$ $= \frac{1}{3} \tan^3 x - \tan x + x + C$

b	$\int \cot^5 x\ dx = \int \cot x \cot^4 x\ dx$ $= \int \cot x (\cot^2 x)^2\ dx$ $= \int \cot x (\csc^2 x - 1)^2\ dx$ $u = \csc x \ \Rightarrow \frac{du}{dx} = -\csc x \cot x \ \Rightarrow dx = \frac{du}{-\csc x \cot x}$ $\Rightarrow \int \cot^5 x\ dx = \int \cot x (u^2 - 1)^2 \times \frac{du}{-\csc x \cot x}$ $= \int (u^2 - 1)^2\ \frac{du}{-u}$ $= \int \frac{u^4 - 2u^2 + 1}{-u}\ du$ $= \int \left(-u^3 + 2u - \frac{1}{u}\right)\ du$ $= -\frac{1}{4}u^4 + u^2 - \ln\|u\| + C$ $= -\frac{1}{4}\csc^4 x + \csc^2 x - \ln\|\csc x\| + C$ حلّ ثانٍ: $\int \cot^5 x\ dx = \int \cot^3 x \cot^2 x\ dx$ $= \int \cot^3 x (\csc^2 x - 1)\ dx$ $= \int \cot^3 x \csc^2 x\ dx - \int \cot^3 x\, dx$ $= \int \cot^3 x \csc^2 x\ dx - \int \cot x \cot^2 x\, dx$ $= \int \cot^3 x \csc^2 x\ dx - \int \cot x (\csc^2 x - 1)\ dx$ $= \int (\cot^3 x - \cot x) \csc^2 x\ dx + \int \cot x\ dx$ في التكامل الأول افرض $u = \cot x$

	$\Rightarrow \frac{du}{dx} = -\csc^2 x \Rightarrow dx = \frac{du}{-\csc^2 x}$ $\int \cot^5 x\ dx = \int (u^3 - u) \csc^2 x \frac{du}{-\csc^2 x} + \int \frac{\cos x}{\sin x} dx$ $= \int (u - u^3)\ du + \int \frac{\cos x}{\sin x} dx$ $= \frac{1}{2}u^2 - \frac{1}{4}u^4 + \ln\lvert\sin x\rvert + C$ $= \frac{1}{2}\cot^2 x - \frac{1}{4}\cot^4 x + \ln\lvert\sin x\rvert + C$
c	$u = \tan x \Rightarrow \frac{du}{dx} = \sec^2 x \Rightarrow dx = \frac{du}{\sec^2 x}$ $\int \sec^4 x \tan^6 x\ dx = \int \sec^4 x\ u^6 \times \frac{du}{\sec^2 x}$ $= \int \sec^2 x\ u^6\ du$ $= \int (1 + \tan^2 x)\ u^6\ du$ $= \int (1 + u^2)\ u^6\ du$ $= \int (u^6 + u^8)\ du$ $= \frac{1}{7}u^7 + \frac{1}{9}u^9 + C$ $= \frac{1}{7}\tan^7 x + \frac{1}{9}\tan^9 x + C$

أتحقق من فهمي صفحة 43	
a	$u = x+1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du,\ x = u-1$ $x = 0 \Rightarrow u = 1$ $x = 2 \Rightarrow u = 3$ $\int_0^2 x(x+1)^3 dx = \int_1^3 (u-1)u^3 du$ $= \int_1^3 (u^4 - u^3) du$ $= \left(\frac{1}{5}u^5 - \frac{1}{4}u^4\right)\Big\|_1^3$ $= \frac{1}{5}(3)^5 - \frac{1}{4}(3)^4 - \left(\frac{1}{5}(1)^5 - \frac{1}{4}(1)^4\right)$ $= \frac{142}{5} = 28.4$
b	$u = \sec x + 2 \Rightarrow \frac{du}{dx} = \sec x \tan x \Rightarrow dx = \frac{du}{\sec x \tan x}$ $x = 0 \Rightarrow u = 3$ $x = \frac{\pi}{3} \Rightarrow u = 4$ $\int_0^{\frac{\pi}{3}} \sec x \tan x \sqrt{\sec x + 2} dx = \int_3^4 \sec x \tan x \sqrt{u} \frac{du}{\sec x \tan x}$ $= \int_3^4 \sqrt{u}\, du$ $= \frac{2}{3}u^{\frac{3}{2}}\Big\|_3^4$ $= \frac{2}{3}(8 - 3\sqrt{3}) \approx 1.87$

أتدرب وأحل المسائل صفحة 44	
1	$u = 2x^3 + 5 \Rightarrow \frac{du}{dx} = 6x^2 \Rightarrow dx = \frac{du}{6x^2}$ $\int x^2(2x^3+5)^4 dx = \int x^2 u^4 \times \frac{du}{6x^2}$ $= \int \frac{1}{6} u^4 \, du$ $= \frac{1}{30} u^5 + C$ $= \frac{1}{30}(2x^3+5)^5 + C$
2	$u = x + 3 \Rightarrow dx = du \quad , x = u - 3$ $\int x^2\sqrt{x+3}\, dx = \int x^2 \sqrt{u}\, du$ $= \int (u-3)^2 \sqrt{u}\, du$ $= \int \left(u^{\frac{5}{2}} - 6u^{\frac{3}{2}} + 9u^{\frac{1}{2}}\right) du$ $= \frac{2}{7}u^{\frac{7}{2}} - \frac{12}{5}u^{\frac{5}{2}} + 6u^{\frac{3}{2}} + C$ $= \frac{2}{7}(x+3)^{\frac{7}{2}} - \frac{12}{5}(x+3)^{\frac{5}{2}} + 6(x+3)^{\frac{3}{2}} + C$ $= \frac{2}{7}\sqrt{(x+3)^7} - \frac{12}{5}\sqrt{(x+3)^5} + 6\sqrt{(x+3)^3} + C$
3	$u = x + 2 \Rightarrow dx = du \quad , x = u - 2$ $\int x(x+2)^3 dx = \int x u^3 du$ $= \int (u-2)u^3\, du$ $= \int (u^4 - 2u^3)\, du$ $= \frac{1}{5}u^5 - \frac{1}{2}u^4 + C$ $= \frac{1}{5}(x+2)^5 - \frac{1}{2}(x+2)^4 + C$

4	$u = x + 4 \quad \Rightarrow dx = du \quad , x = u - 4$ $\int \frac{x}{\sqrt{x+4}} dx = \int \frac{x}{\sqrt{u}} du$ $= \int \frac{u-4}{\sqrt{u}} du$ $= \int \left(u^{\frac{1}{2}} - 4u^{-\frac{1}{2}}\right) du$ $= \frac{2}{3} u^{\frac{3}{2}} - 8u^{\frac{1}{2}} + C$ $= \frac{2}{3} (x+4)^{\frac{3}{2}} - 8(x+4)^{\frac{1}{2}} + C$ $= \frac{2}{3} \sqrt{(x+4)^3} - 8\sqrt{x+4} + C$
5	$\int \sin x \cos 2x \, dx = \int \sin x \, (2\cos^2 x - 1) dx$ $u = \cos x \quad \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$ $\int \sin x \cos 2x \, dx = \int \sin x \, (2u^2 - 1) \times \frac{du}{-\sin x}$ $= \int (1 - 2u^2) du$ $= u - \frac{2}{3} u^3 + C$ $= \cos x - \frac{2}{3} \cos^3 x + C$

6	$u = e^x + 1 \Rightarrow \frac{du}{dx} = e^x \Rightarrow dx = \frac{du}{e^x} \quad , e^x = u - 1$ $\int \frac{e^{3x}}{e^x+1} dx = \int \frac{e^{3x}}{u} \times \frac{du}{e^x}$ $= \int \frac{e^{2x}}{u} du$ $= \int \frac{(u-1)^2}{u} du$ $= \int \left(u - 2 + \frac{1}{u}\right) du$ $= \frac{1}{2}u^2 - 2u + \ln\lvert u\rvert + C$ $= \frac{1}{2}(e^x+1)^2 - 2(e^x+1) + \ln(e^x+1) + C$
7	$\int \sec^4 x\, dx = \int \sec^2 x \times \sec^2 x\, dx = \int \sec^2 x\,(1 + \tan^2 x) dx$ $u = \tan x \Rightarrow \frac{du}{dx} = \sec^2 x \Rightarrow dx = \frac{du}{\sec^2 x}$ $\int \sec^4 x\, dx = \int \sec^2 x\,(1+u^2) \times \frac{du}{\sec^2 x}$ $= \int (1+u^2)\, du$ $= u + \frac{1}{3}u^3 + C$ $= \tan x + \frac{1}{3}\tan^3 x + C$

8	$\int \frac{\tan x}{\cos^2 x} dx = \int \tan x \sec^2 x\, dx$ $u = \tan x \Rightarrow \frac{du}{dx} = \sec^2 x \Rightarrow dx = \frac{du}{\sec^2 x}$ $\int \frac{\tan x}{\cos^2 x} dx = \int u \sec^2 x \times \frac{du}{\sec^2 x}$ $= \int u\, du$ $= \frac{1}{2} u^2 + C$ $= \frac{1}{2} \tan^2 x + C$
9	$u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\, du$ $\int \frac{\sin(\ln x)}{x} dx = \int \frac{\sin u}{x} \times x\, du$ $= \int \sin u\, du$ $= -\cos u + C$ $= -\cos(\ln x) + C$
10	$\int \frac{\sin x \cos x}{1 + \sin^2 x} dx = \frac{1}{2} \int \frac{2 \sin x \cos x}{1 + \sin^2 x} dx$ $= \frac{1}{2} \ln(1 + \sin^2 x) + C$
11	$u = e^x + e^{-x} \Rightarrow \frac{du}{dx} = e^x - e^{-x} \Rightarrow dx = \frac{du}{e^x - e^{-x}}$ $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx = \int \frac{2(e^x - e^{-x})}{u^2} \times \frac{du}{e^x - e^{-x}}$ $= \int 2u^{-2} du$ $= -2u^{-1} + C$ $= -\frac{2}{e^x + e^{-x}} + C$

12	$u = x + 1 \quad \Rightarrow dx = du\,,\; x = u - 1$ $\int \frac{-x}{(x+1)\sqrt{x+1}}dx = \int \frac{1-u}{u\sqrt{u}}du$ $= \int \frac{1-u}{u^{\frac{3}{2}}}du$ $= \int \left(u^{-\frac{3}{2}} - u^{-\frac{1}{2}}\right)du$ $= -2u^{-\frac{1}{2}} - 2u^{\frac{1}{2}} + C$ $= -2(x+1)^{-\frac{1}{2}} - 2(x+1)^{\frac{1}{2}} + C$ $= \frac{-2}{\sqrt{x+1}} - 2\sqrt{x+1} + C$
13	$u = x + 10 \quad \Rightarrow dx = du \quad , x = u - 10$ $\int x\sqrt[3]{x+10}\,dx = \int (u-10)u^{\frac{1}{3}}\,du$ $= \int \left(u^{\frac{4}{3}} - 10u^{\frac{1}{3}}\right)du$ $= \frac{3}{7}u^{\frac{7}{3}} - \frac{15}{2}u^{\frac{4}{3}} + C$ $= \frac{3}{7}(x+10)^{\frac{7}{3}} - \frac{15}{2}(x+10)^{\frac{4}{3}} + C$ $= \frac{3}{7}\sqrt[3]{(x+10)^7} - \frac{15}{2}\sqrt[3]{(x+10)^4} + C$

14	$u = \tan\frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2}\sec^2\frac{x}{2} \Rightarrow dx = \frac{2du}{\sec^2\frac{x}{2}}$ $\int \sec^2\frac{x}{2}\tan^7\frac{x}{2}\,dx = \int \sec^2\frac{x}{2}\,u^7 \times \frac{2du}{\sec^2\frac{x}{2}}$ $= 2\int u^7\,du$ $= \frac{1}{4}u^8 + C$ $= \frac{1}{4}\tan^8\frac{x}{2} + C$
15	$\int \frac{\sec^3 x + e^{\sin x}}{\sec x}dx = \int (\sec^2 x + \cos x\, e^{\sin x})dx$ $= \int \sec^2 x\,dx + \int \cos x\, e^{\sin x}dx$ $u = \sin x \Rightarrow \frac{du}{dx} = \cos x \Rightarrow dx = \frac{du}{\cos x}$ $\int \frac{\sec^3 x + e^{\sin x}}{\sec x}dx = \int \sec^2 x\,dx + \int \cos x\, e^u \times \frac{du}{\cos x}$ $= \tan x + \int e^u du$ $= \tan x + e^u + C$ $= \tan x + e^{\sin x} + C$

16	$u = \sin x \Rightarrow \frac{du}{dx} = \cos x \Rightarrow dx = \frac{du}{\cos x}$ $\int (1 + \sqrt[3]{\sin x}) \cos^3 x \, dx = \int \left(1 + u^{\frac{1}{3}}\right) \cos^3 x \, \frac{du}{\cos x}$ $= \int \left(1 + u^{\frac{1}{3}}\right) \cos^2 x \, du$ $= \int \left(1 + u^{\frac{1}{3}}\right) (1 - \sin^2 x) \, du$ $= \int \left(1 + u^{\frac{1}{3}}\right) (1 - u^2) \, du$ $= \int \left(1 + u^{\frac{1}{3}}\right) (1 - u^2) \, du$ $= \int \left(1 - u^2 + u^{\frac{1}{3}} - u^{\frac{7}{3}}\right) du$ $= u - \frac{1}{3}u^3 + \frac{3}{4}u^{\frac{4}{3}} - \frac{3}{10}u^{\frac{10}{3}} + C$ $= \sin x - \frac{1}{3}\sin^3 x + \frac{3}{4}\sin^{\frac{4}{3}} x - \frac{3}{10}\sin^{\frac{10}{3}} x + C$

17	$\int \sin x \sec^5 x\, dx = \int \sin x \cos^{-5} x\, dx$ $u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$ $\int \sin x \sec^5 x\, dx = \int \sin x\, u^{-5} \times \frac{du}{-\sin x}$ $= -\int u^{-5}\, du$ $= \frac{1}{4} u^{-4} + C$ $= \frac{1}{4} \cos^{-4} x + C$ $= \frac{1}{4} \sec^4 x + C$
18	$\int \frac{\sin x + \tan x}{\cos^3 x}\, dx = \int (\tan x \sec^2 x + \tan x \sec^3 x)\, dx$ $= \int \tan x \sec x (\sec x + \sec^2 x)\, dx$ $u = \sec x \Rightarrow \frac{du}{dx} = \tan x \sec x \Rightarrow dx = \frac{du}{\tan x \sec x}$ $\int \frac{\sin x + \tan x}{\cos^3 x}\, dx = \int \tan x \sec x (u + u^2) \frac{du}{\tan x \sec x}$ $= \int (u + u^2)\, du$ $= \frac{1}{2} u^2 + \frac{1}{3} u^3 + C$ $= \frac{1}{2} \sec^2 x + \frac{1}{3} \sec^3 x + C$

19	$\sqrt{1-\cos^2 2x} = \sqrt{\sin^2 2x} = \lvert\sin 2x\rvert$ لكن الزاوية 2x تكون ضمن الربع الأول عندما $0 < x < \frac{\pi}{4}$ لذا فإن $\sin 2x > 0$ ويكون $\lvert\sin 2x\rvert = \sin 2x$ $\int_0^{\frac{\pi}{4}} \sin x \sqrt{1-\cos^2 2x}\, dx = \int_0^{\frac{\pi}{4}} \sin x \sin 2x \, dx$ $= \int_0^{\frac{\pi}{4}} 2\sin^2 x \cos x \, dx$ $u = \sin x \implies \frac{du}{dx} = \cos x \implies dx = \frac{du}{\cos x}$ $x = 0 \implies u = 0$ $x = \frac{\pi}{4} \implies u = \frac{1}{\sqrt{2}}$ $\int_0^{\frac{\pi}{4}} \sin x \sqrt{1-\cos^2 2x}\, dx = \int_0^{\frac{1}{\sqrt{2}}} 2u^2 \cos x \frac{du}{\cos x}$ $= \int_0^{\frac{1}{\sqrt{2}}} 2u^2 du$ $= \frac{2}{3}u^3 \Big\vert_0^{\frac{1}{\sqrt{2}}}$ $= \frac{1}{3\sqrt{2}}$

20	$u = x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ $x = \frac{\pi}{2} \Rightarrow u = \frac{\pi^2}{4}$ $x = 0 \Rightarrow u = 0$ $\int_0^{\frac{\pi}{2}} x \sin x^2 \, dx = \int_0^{\frac{\pi^2}{4}} x \sin u \frac{du}{2x}$ $= \frac{1}{2}\int_0^{\frac{\pi^2}{4}} \sin u \, du$ $= -\frac{1}{2}\cos u \Big\|_0^{\frac{\pi^2}{4}}$ $= -\frac{1}{2}\left(\cos\frac{\pi^2}{4} - 1\right) \approx 0.891$
21	$u = 1 + x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$, $x^2 = u - 1$ $x = 0 \Rightarrow u = 1$ $x = 1 \Rightarrow u = 2$ $\int_0^1 \frac{x^3}{\sqrt{1+x^2}} dx = \int_1^2 \frac{x^3}{\sqrt{u}} \times \frac{du}{2x} = \frac{1}{2}\int_1^2 \frac{x^2}{\sqrt{u}} du = \frac{1}{2}\int_1^2 \frac{u-1}{\sqrt{u}} du$ $= \frac{1}{2}\int_1^2 \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$ $= \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right)\Big\|_1^2$ $= \frac{1}{2}\left(\frac{2}{3}(2)^{\frac{3}{2}} - 2(2)^{\frac{1}{2}} - (\frac{2}{3}(1) - 2(1))\right)$ $= \frac{2 - \sqrt{2}}{3}$

22	$u = \tan x \Rightarrow \frac{du}{dx} = \sec^2 x \Rightarrow dx = \frac{du}{\sec^2 x}$ $x = 0 \Rightarrow u = 0$ $x = \frac{\pi}{3} \Rightarrow u = \sqrt{3}$ $\int_0^{\frac{\pi}{3}} \sec^2 x \tan^5 x\, dx = \int_0^{\sqrt{3}} \sec^2 x\, u^5 \frac{du}{\sec^2 x}$ $= \int_0^{\sqrt{3}} u^5\, du$ $= \frac{1}{6} u^6 \Big\|_0^{\sqrt{3}}$ $= \frac{9}{2}$
23	$u = (x-1)^2 \Rightarrow \frac{du}{dx} = 2(x-1) \Rightarrow dx = \frac{du}{2(x-1)}$ $x = 0 \Rightarrow u = 1$ $x = 2 \Rightarrow u = 1$ $\int_0^2 (x-1)e^{(x-1)^2} dx = \int_1^1 (x-1)e^u \frac{du}{2(x-1)} = 0$
24	$u = 2 + \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dx = 2\sqrt{x}\, du$ $x = 1 \Rightarrow u = 3$ $x = 4 \Rightarrow u = 4$ $\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}} dx = \int_3^4 \frac{\sqrt{u}}{\sqrt{x}} 2\sqrt{x}\, du = \int_3^4 2\sqrt{u}\, du = \frac{4}{3} u^{\frac{3}{2}} \Big\|_3^4 = \frac{4(8-3\sqrt{3})}{3}$

25	$u = 1 + x^{\frac{3}{2}} \Rightarrow \frac{du}{dx} = \frac{3}{2}x^{\frac{1}{2}} \Rightarrow dx = \frac{2}{3}\frac{du}{x^{\frac{1}{2}}}$ $x = 0 \Rightarrow u = 1$ $x = 1 \Rightarrow u = 2$ $\int_0^1 \frac{10\sqrt{x}}{(1+\sqrt{x^3})^2}dx = \int_1^2 \frac{10\sqrt{x}}{u^2}\frac{2}{3}\frac{du}{x^{\frac{1}{2}}}$ $= \frac{20}{3}\int_1^2 u^{-2}\,du = -\frac{20}{3}u^{-1}\Big\|_1^2 = \frac{10}{3}$
26	$u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$ $x = 0 \Rightarrow u = 1$ $x = \frac{\pi}{6} \Rightarrow u = \frac{\sqrt{3}}{2}$ $\int_0^{\frac{\pi}{6}} 2^{\cos x}\sin x\ dx = \int_1^{\frac{\sqrt{3}}{2}} 2^u \sin x \frac{du}{-\sin x}$ $= -\int_1^{\frac{\sqrt{3}}{2}} 2^u\ du$ $= -\frac{2^u}{\ln 2}\Big\|_1^{\frac{\sqrt{3}}{2}}$ $= -\frac{1}{\ln 2}\left(2^{\frac{\sqrt{3}}{2}} - 2\right) \approx 0.256$

27	$u = \cot x \Rightarrow \frac{du}{dx} = -\csc^2 x \Rightarrow dx = \frac{du}{-\csc^2 x}$ $x = \frac{\pi}{2} \Rightarrow u = 0$ $x = \frac{\pi}{4} \Rightarrow u = 1$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2 x \cot^5 x\, dx = \int_1^0 \csc^2 x\, u^5 \frac{du}{-\csc^2 x}$ $= \int_1^0 -u^5\, du$ $= -\frac{1}{6}u^6 \Big\|_1^0$ $= \frac{1}{6}$
28	$A = -\int_{-1}^0 6x(x^2+1)^3 dx + \int_0^1 6x(x^2+1)^3 dx$ $u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ $x = -1 \Rightarrow u = 2$ $x = 0 \Rightarrow u = 1$ $x = 1 \Rightarrow u = 2$ $A = -\int_2^1 6xu^3 \frac{du}{2x} + \int_1^2 6xu^3 \frac{du}{2x}$ $= \int_1^2 3u^3 du + \int_1^2 3u^3 du = \int_1^2 6u^3 du$ $= \frac{6}{4}u^4 \Big\|_1^2$ $= \frac{45}{2}$

29	$A = \int_2^4 \frac{x}{(x-1)^3} dx$ $u = x - 1 \quad \Rightarrow dx = du \quad , \quad x = u + 1$ $x = 2 \quad \Rightarrow u = 1$ $x = 4 \quad \Rightarrow u = 3$ $A = \int_2^4 \frac{x}{(x-1)^3} dx = \int_1^3 \frac{u+1}{u^3} du$ $= \int_1^3 (u^{-2} + u^{-3}) du = \left(-u^{-1} - \frac{1}{2}u^{-2}\right)\Big\|_1^3$ $= -\frac{1}{3} - \frac{1}{2}\left(\frac{1}{9}\right) + 1 + \frac{1}{2}$ $= \frac{10}{9}$
30	$u = x^2 \quad \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ $x = -1 \quad \Rightarrow u = 1$ $x = 0 \quad \Rightarrow u = 0$ $x = 2 \quad \Rightarrow u = 4$ $A = -\int_{-1}^0 xe^{x^2} dx + \int_0^2 xe^{x^2} dx = -\int_1^0 xe^u \frac{du}{2x} + \int_0^4 xe^u \frac{du}{2x}$ $= -\int_1^0 \frac{1}{2} e^u du + \int_0^4 \frac{1}{2} e^u du$ $= -\frac{1}{2} e^u \Big\|_1^0 + \frac{1}{2} e^u \Big\|_0^4$ $= -\frac{1}{2} e^0 + \frac{1}{2} e^1 + \frac{1}{2} e^4 - \frac{1}{2} e^0$ $= \frac{1}{2}(e^4 + e) - 1 \approx 27.658$

31	$u = x^2 + \frac{\pi}{6} \quad \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ $x = \sqrt{\frac{\pi}{6}} \quad \Rightarrow u = \frac{\pi}{3}$ $x = 0 \quad \Rightarrow u = \frac{\pi}{6}$ $A = \int_0^{\sqrt{\frac{\pi}{6}}} 2x \cos\left(x^2 + \frac{\pi}{6}\right) dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2x \cos u \frac{du}{2x}$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos u \, du$ $= \sin u \Big\vert_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $= \sin\frac{\pi}{3} - \sin\frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2} \approx 0.366$
32	$f(x) = \int f'(x)\, dx = \int 2x(4x^2 - 10)^2\, dx$ $u = 4x^2 - 10 \Rightarrow \frac{du}{dx} = 8x \Rightarrow dx = \frac{du}{8x}$ $f(x) = \int 2xu^2 \frac{du}{8x} = \int u^2 \frac{du}{4}$ $= \frac{1}{4}\int u^2\, du$ $= \frac{1}{12}u^3 + C$ $\Rightarrow f(x) = \frac{1}{12}(4x^2 - 10)^3 + C$ $f(2) = \frac{1}{12}(216) + C = 10 \Rightarrow C = -8$ $10 = 18 + C \quad \Rightarrow C = -8$ $\Rightarrow f(x) = \frac{1}{12}(4x^2 - 10)^3 - 8$

33	$f(x)=\int f'(x)\,dx=\int x^2 e^{-0.2x^3}dx$ $u=-0.2x^3 \Rightarrow \frac{du}{dx}=-0.6x^2 \Rightarrow dx=\frac{du}{-0.6x^2}$ $f(x)=\int x^2 e^u \frac{du}{-0.6x^2}=-\frac{10}{6}\int e^u\,du$ $=-\frac{5}{3}e^u+C$ $\Rightarrow f(x)=-\frac{5}{3}e^{-0.2x^3}+C$ $f(0)=-\frac{5}{3}+C$ $\frac{3}{2}=-\frac{5}{3}+C \Rightarrow C=\frac{19}{6}$ $\Rightarrow f(x)=-\frac{5}{3}e^{-0.2x^3}+\frac{19}{6}$
34	نجد أصفار الاقتران بحل المعادلة $f(x)=0$ $x(x-2)^4=0 \Rightarrow x=0\ ,x=2$ نقطة التقاطع $(0,0)$ ، فتكون نقطة التماس $(2,0)$ ويمكن التحقق بحساب $f'(2)$: $f'(x)=(x-2)^4+4x(x-2)^3$ $f'(2)=(2-2)^4+4(2)(2-2)^3=0$

35	$A = \int_0^2 x(x-2)^4\, dx$ $u = x - 2 \;\Rightarrow dx = du \quad , \quad x = u + 2$ $x = 0 \;\Rightarrow u = -2$ $x = 2 \;\Rightarrow u = 0$ $A = \int_0^2 x(x-2)^4\, dx = \int_{-2}^0 (u+2)u^4\, du$ $= \int_{-2}^0 (u^5 + 2u^4)\, du = \left(\frac{1}{6}u^6 + \frac{2}{5}u^5\right)\Big\|_{-2}^0$ $= 0 - \left(\frac{1}{6}(-2)^6 + \frac{2}{5}(-2)^5\right) = \frac{32}{15}$
36	$s(t) = \int \sin\omega t\ \cos^2\omega t\, dt$ $u = \cos\omega t \Rightarrow \frac{du}{dx} = -\omega\sin\omega t \Rightarrow dt = \frac{du}{-\omega\sin\omega t}$ $s(t) = \int \sin\omega t\ u^2 \frac{du}{-\omega\sin\omega t}$ $= \frac{-1}{\omega}\int u^2\, du = \frac{-1}{3\omega}u^3 + C$ $\Rightarrow s(t) = -\frac{1}{3\omega}\cos^3\omega t + C$ لكن $s(0) = 0$ لأن الجسيم انطلق من نقطة الأصل. $s(0) = -\frac{1}{3\omega} + C$ $0 = -\frac{1}{3\omega} + C \;\Rightarrow C = \frac{1}{3\omega}$ $\Rightarrow s(t) = -\frac{1}{3\omega}\cos^3\omega t + \frac{1}{3\omega}$

37	$C(t) = \int C'(t)dt = \int \frac{-0.01e^{-0.01t}}{(1+e^{-0.01t})^2}dt$ $u = 1 + e^{-0.01t} \Rightarrow \frac{du}{dt} = -0.01e^{-0.01t} \Rightarrow dt = \frac{du}{-0.01e^{-0.01t}}$ $C(t) = \int \frac{-0.01e^{-0.01t}}{u^2} \times \frac{du}{-0.01e^{-0.01t}} = \int u^{-2}\, du$ $= -u^{-1} + K$ (استعمل الرمز K لثابت التكامل بدل C المعتاد لتمييز ثابت التكامل عن رمز الاقتران C). $C(t) = -(1+e^{-0.01t})^{-1} + K$ $C(0) = -(2)^{-1} + K$ $\frac{1}{2} = -\frac{1}{2} \Rightarrow K = 1$ $\Rightarrow C(t) = -(1+e^{-0.01t})^{-1} + 1$ $C(t) = \frac{-1}{1+e^{-0.01t}} + 1$

38	$u = e^x - 2 \Rightarrow \frac{du}{dx} = e^x \Rightarrow dx = \frac{du}{e^x}$ $e^x = u + 2$ $x = \ln 3 \Rightarrow u = e^{\ln 3} - 2 = 3 - 2 = 1$ $x = \ln 4 \Rightarrow u = e^{\ln 4} - 2 = 4 - 2 = 2$ $\int_{\ln 3}^{\ln 4} \frac{e^{4x}}{e^x - 2}\, dx = \int_1^2 \frac{e^{4x}}{u} \frac{du}{e^x} = \int_1^2 \frac{e^{3x}}{u}\, du$ $= \int_1^2 \frac{(u+2)^3}{u}\, du$ $= \int_1^2 \frac{u^3 + 6u^2 + 12u + 8}{u}\, du$ $= \int_1^2 \left(u^2 + 6u + 12 + \frac{8}{u}\right) du$ $= \left(\frac{1}{3}u^3 + 3u^2 + 12u + 8\ln\|u\|\right)\Big\|_1^2$ $= \left(\frac{1}{3}(2)^3 + 3(2)^2 + 12(2) + 8\ln 2\right) - \left(\frac{1}{3}(1)^3 + 3(1)^2 + 12(1) + 8\ln 1\right)$ $= \frac{70}{3} + 8\ln 2$
39	$f(x) = \int \tan x\, dx = -\int \frac{-\sin x}{\cos x}\, dx = -\ln\|\cos x\| + C$ $f(3) = -\ln\|\cos 3\| + C$ $5 = -\ln\|\cos 3\| + C \Rightarrow C = 5 + \ln\|\cos 3\|$ $f(x) = -\ln\|\cos x\| + 5 + \ln\|\cos 3\| = \ln\left\|\frac{\cos 3}{\cos x}\right\| + 5$

40	$f(x) = 0 \implies 3\cos x\sqrt{1+\sin x} = 0$ $\cos x = 0 \implies x = \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}, x = \frac{3\pi}{2} + 2n\pi, n \in \mathbb{Z}$ $\sin x = -1 \implies x = \frac{3\pi}{2} + 2n\pi, n \in \mathbb{Z}$ يوجد عدد لا نهائي من الحلول لهاتين المعادلتين، ونريد أصغر حلين موجبين (الإحداثي x للنقطتين C,B) وأكبر حل سالب (الإحداثي x للنقطة A) أصغر حلين موجبين هما: $x = \frac{\pi}{2}, x = \frac{3\pi}{2}$ بوضع n = 0 $\implies B\left(\frac{\pi}{2}, 0\right)$, $C\left(\frac{3\pi}{2}, 0\right)$ أكبر حل سالب هو: $x = -\frac{\pi}{2}$، بوضع n = -1 $\implies A\left(-\frac{\pi}{2}, 0\right)$ أما النقطة D فإحداثياها هما: $D(0, f(0)) = (0, 3)$
41	$A = A_1 + A_2 = A(R_1) + A(R_2)$ $A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3\cos x\sqrt{1+\sin x})\, dx + \left(-\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (3\cos x\sqrt{1+\sin x})\, dx\right)$ $u = 1 + \sin x \implies \frac{du}{dx} = \cos x \implies dx = \frac{du}{\cos x}$ $x = -\frac{\pi}{2} \implies u = 0$ $x = \frac{\pi}{2} \implies u = 2$ $x = \frac{3\pi}{2} \implies u = 0$ $A = 3\int_0^2 \cos x\sqrt{u}\, \frac{du}{\cos x} + \left(-3\int_2^0 \cos x\sqrt{u}\, \frac{du}{\cos x}\right)$ $= 3\int_0^2 \sqrt{u}\, du + 3\int_0^2 \sqrt{u}\, du$ $= 6\int_0^2 \sqrt{u}\, du = 4u^{\frac{3}{2}}\Big\vert_0^2 = 4(2\sqrt{2} - 0) = 8\sqrt{2}$

من حل السؤال السابق نجد أن:

42	$A(R_1) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3\cos x\sqrt{1+\sin x})\,dx = \int_0^2 3\sqrt{u}\,du = 4\sqrt{2}$ $A(R_2) = -\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (3\cos x\sqrt{1+\sin x})dx = -\int_2^0 3\sqrt{u}\,du = 4\sqrt{2}$ $\Rightarrow A(R_1) = A(R_2)$
43	$u = 1 + x^{\frac{3}{4}} \Rightarrow \frac{du}{dx} = \frac{3}{4}x^{-\frac{1}{4}} \Rightarrow dx = \frac{4}{3}x^{\frac{1}{4}}du, \quad x^{\frac{3}{4}} = u - 1$ $x = 1 \Rightarrow u = 2$ $x = 16 \Rightarrow u = 9$ $\int_1^{16} \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}dx = \int_2^9 \frac{x^{\frac{1}{2}}}{u}\frac{4}{3}x^{\frac{1}{4}}du$ $= \frac{4}{3}\int_2^9 \frac{x^{\frac{3}{4}}}{u}du = \frac{4}{3}\int_2^9 \frac{u-1}{u}du = \frac{4}{3}\int_2^9 \left(1-\frac{1}{u}\right)du$ $= \frac{4}{3}(u - \ln\|u\|)\Big\|_2^9 = \frac{4}{3}\left(7 - \ln\frac{9}{2}\right)$
44	$\int_0^{\frac{\pi}{2}} f(\cos x)\,dx = \int_0^{\frac{\pi}{2}} f\left(\sin\left(\frac{\pi}{2} - x\right)\right)dx$ $u = \frac{\pi}{2} - x \Rightarrow dx = -du$ $x = 0 \Rightarrow u = \frac{\pi}{2}$ $x = \frac{\pi}{2} \Rightarrow u = 0$ $\int_0^{\frac{\pi}{2}} f(\cos x)\,dx = \int_{\frac{\pi}{2}}^0 -f(\sin u)\,du = \int_0^{\frac{\pi}{2}} f(\sin u)\,du = \int_0^{\frac{\pi}{2}} f(\sin x)\,dx$

45	$u = 1 - x \quad \Rightarrow dx = -du \quad , x = 1 - u$ $x = 0 \quad \Rightarrow u = 1$ $x = 1 \quad \Rightarrow u = 0$ $\int_0^1 x^a(1-x)^b\, dx = \int_1^0 -(1-u)^a u^b\, du$ $= \int_0^1 u^b(1-u)^a\, du = \int_0^1 x^b(1-x)^a\, dx$
46	$u = \ln(\ln x) \Rightarrow \frac{du}{dx} = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x\ln x} \Rightarrow dx = x\ln x\, du$ $\int \frac{dx}{x\ln x\,(\ln(\ln x))} = \int \frac{x\ln x\, du}{ux\ln x} = \ln\|u\| + C = \ln\|\ln(\ln x)\| + C$
47	$u = \sin x + \cos x \Rightarrow \frac{du}{dx} = \cos x - \sin x \Rightarrow dx = \frac{du}{\cos x - \sin x}$ $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx = \int \frac{\sin x - \cos x}{u} \times \frac{du}{\cos x - \sin x}$ $= \int \frac{-(\cos x - \sin x)}{u} \times \frac{du}{\cos x - \sin x}$ $= -\int \frac{1}{u} du = -\ln\|u\| + C$ $= -\ln\|\sin x + \cos x\| + C$
48	$u = 1 + \sin x \quad \Rightarrow \frac{du}{dx} = \cos x \quad \Rightarrow dx = \frac{du}{\cos x} \quad , \quad \sin x = u - 1$ $\int \sin 2x\,(1 + \sin x)^3 dx = \int 2\sin x \cos x\, u^3 \frac{du}{\cos x}$ $= \int 2(u-1)u^3 du$ $= \int (2u^4 - 2u^3) du$ $= \frac{2}{5}u^5 - \frac{1}{2}u^4 + C$ $= \frac{2}{5}(1 + \sin x)^5 - \frac{1}{2}(1 + \sin x)^4 + C$