الرياضيات

الصف الثاني عشر- الفرع العلمي

الفصل الدراسي الثاني

12

إجابات الطالب

منهاجي
متعة التعليم الهادف

الناشر: المركز الوطني لتطوير المناهج

يسر المركز الوطني لتطوير المناهج استقبال آرائكم وملحوظاتكم على هذا الكتاب عن طريق العناوين الآتية:

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إجابات كتاب الطالب للصف الثاني عشر العلمي / الفصل الدراسي الثاني

الوحدة الرابعة: التكامل

الدرس الأول: تكاملات اقترانات خاصة

مسألة اليوم صفحة 8	
	$P(t)=\int(200e^{0.1t}+150e^{-0.03t})dt=\frac{200}{0.1}e^{0.1t}+\frac{150}{-0.03}e^{-0.03t}+C$ $=2000e^{0.1t}-5000e^{-0.03t}+C$ $P(0)=2000-5000+C$ $200000=\ -3000+C\Rightarrow C=203000$ $P(t)=2000e^{0.1t}-5000e^{-0.03t}+203000$ $P(12)=2000e^{1.2}-5000e^{-0.36}+203000\approx 206152$ إذن، سيكون عدد الخلايا بعد 12 يومًا 206152 خلية تقريبًا.
أتحقق من فهمي صفحة 10	
a	$\int(5x^2-3e^{7x})dx=\frac{5}{3}x^3-\frac{3}{7}e^{7x}+C$
b	$\int_0^{\ln 3}8e^{4x}\,dx=\frac{8}{4}e^{4x}\Big\|_0^{\ln 3}=2\left(e^{4\ln 3}-e^0\right)=2\left(e^{\ln 3^4}-e^0\right)$ $=2(81-1)=160$
C	$\int\sqrt{e^{1-x}}dx=\int(e^{1-x})^{1/2}dx=\int e^{(1-x)/2}dx=-2e^{(1-x)/2}+C$
d	$\int(3^x+2\sqrt{x})dx=\frac{3^x}{\ln 3}+2(\frac{2}{3}x^{\frac{3}{2}})+C=\frac{3^x}{\ln 3}+\frac{4}{3}x^{\frac{3}{2}}+C$
أتحقق من فهمي صفحة 12	
a	$\int\cos(3x-\pi)\,dx=\frac{1}{3}\sin(3x-\pi)+C$
b	$\int(\csc^2(5x)+e^{2x})dx=-\frac{1}{5}\cot 5x+\frac{1}{2}e^{2x}+C$

c	$\int_0^{\frac{\pi}{3}} (\sin 2x - \cos 4x)\, dx = \left(-\frac{1}{2}\cos 2x - \frac{1}{4}\sin 4x\right)\Big\|_0^{\frac{\pi}{3}}$ $= \left(-\frac{1}{2}\cos\frac{2\pi}{3} - \frac{1}{4}\sin\frac{4\pi}{3}\right) - \left(-\frac{1}{2}\cos 0 - \frac{1}{4}\sin 0\right)$ $= \left(-\frac{1}{2}(-\frac{1}{2}) - \frac{1}{4}(\frac{-\sqrt{3}}{2})\right) - \left(-\frac{1}{2} - 0\right)$ $= \frac{6+\sqrt{3}}{8}$
أتحقق من فهمي صفحة 14	
a	$\int \cos^4 x\, dx$ $\cos^4 x = (\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2$ $= \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)$ $= \frac{1}{4}\left(1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right)$ $= \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x$ $= \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$ $\int \cos^4 x\, dx = \int \left(\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x\right) dx$ $= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

b	$\int_0^{\frac{\pi}{6}} \sin 3x \sin x \, dx = \int_0^{\frac{\pi}{6}} \frac{1}{2}(\cos(3x-x) - \cos(3x+x))\, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{6}} (\cos 2x - \cos 4x)\, dx$ $= \left(\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x\right)\Big\|_0^{\frac{\pi}{6}}$ $= \left(\frac{1}{4}\sin\frac{2\pi}{6} - \frac{1}{8}\sin\frac{4\pi}{6}\right) - (0-0) = \frac{\sqrt{3}}{8} - \frac{\sqrt{3}}{16} = \frac{\sqrt{3}}{16}$
c	$\int \frac{dx}{1+\cos x} = \int (\frac{1}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}) dx$ $= \int \frac{1-\cos x}{\sin^2 x} dx$ $= \int (\csc^2 x - \cot x \csc x) dx$ $= -\cot x + \csc x + C$
أتحقق من فهمي صفحة 16	
a	$\int \left(\sin x - \frac{5}{x}\right) dx = -\cos x - 5\ln\|x\| + C$
b	$\int \frac{5}{3x+2} dx = \frac{5}{3}\int \frac{3}{3x+2} dx = \frac{5}{3}\ln\|3x+2\| + C$
c	$\int \frac{x^2-7x+2}{x^2} dx = \int \left(1 - \frac{7}{x} + 2x^{-2}\right) dx = x - 7\ln\|x\| - 2x^{-1} + C$
d	$\int \frac{2x+3}{x^2+3x} dx = \ln\|x^2+3x\| + C$
e	$\int \frac{\sin 2x}{1+\cos 2x} dx = -\frac{1}{2}\int \frac{-2\sin 2x}{1+\cos 2x} dx$ $= -\frac{1}{2}\ln\|1+\cos 2x\| + C$ $= -\frac{1}{2}\ln(1+\cos 2x) + C$

f	$\int \cot x\, dx = \int \frac{\cos x}{\sin x} dx = \ln\lvert\sin x\rvert + C$
g	$\int \frac{e^x}{e^x+7} dx = \ln\lvert e^x+7\rvert + C = \ln(e^x+7) + C$
h	$\int \csc x\, dx = \int \csc x \times \frac{\csc x + \cot x}{\csc x + \cot x} dx$ $= \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} dx$ $= -1\int \frac{-\csc^2 x - \csc x \cot x}{\csc x + \cot x} dx = -\ln\lvert\csc x + \cot x\rvert + C$

أتحقق من فهمي صفحة 17

$$\int \frac{x^2+x+1}{x+1} dx = \int \left(x + \frac{1}{x+1}\right) dx = \frac{1}{2}x^2 + \ln\lvert x+1\rvert + C$$

أتحقق من فهمي صفحة 19

a	$\int_{-1}^{3} f(x)\, dx = \int_{-1}^{1} (1+x)\, dx + \int_{1}^{3} 2x\, dx$ $= \left(x + \frac{1}{2}x^2\right)\Big\|_{-1}^{1} + x^2\Big\|_{1}^{3}$ $= \left(1+\frac{1}{2}\right) - \left(-1+\frac{1}{2}\right) + 9 - 1 = 10$
b	$f(x) = \begin{cases} 1-x, & x \le 1 \\ x-1, & x > 1 \end{cases}$ $\int_{-2}^{2} f(x)\, dx = \int_{-2}^{1} (1-x)\, dx + \int_{1}^{2} (x-1)\, dx$ $= \left(x - \frac{1}{2}x^2\right)\Big\|_{-2}^{1} + \left(\frac{1}{2}x^2 - x\right)\Big\|_{1}^{2}$ $= \left(1-\frac{1}{2}\right) - (-2-2) + (2-2) - \left(\frac{1}{2} - 1\right) = 5$

c	$f(x)=\begin{cases} x^2-1, x<-1 \\ 1-x^2, -1\le x\le 1 \\ x^2-1, x>1 \end{cases}$ $\int_{-4}^{0} f(x)\,dx=\int_{-4}^{-1}(x^2-1)\,dx+\int_{-1}^{0}(1-x^2)\,dx$ $=\left(\frac{1}{3}x^3-x\right)\Big\|_{-4}^{-1}+\left(x-\frac{1}{3}x^3\right)\Big\|_{-1}^{0}$ $=\left(-\frac{1}{3}+1\right)-\left(-\frac{64}{3}+4\right)+(0-0)-\left(-1+\frac{1}{3}\right)$ $=\frac{56}{3}$
أتحقق من فهمي صفحة 20	
	$R(t)=\int \frac{21}{0.07t+5}dt$ $=\frac{21}{0.07}\int \frac{0.07}{0.07t+5}dt=300\ln\|0.07t+5\|+C$ $R(0)=300\ln 5+C$ $0=300\ln 5+C \Rightarrow C=-300\ln 5$ $R(t)=300\ln\|0.07t+5\|-300\ln 5=300\ln\left\|\frac{0.07t+5}{5}\right\|$ $=300\ln\|0.014t+1\|$
أتحقق من فهمي صفحة 23	
a	$s(t)=\int v(t)dt=\int 3\cos t\,dt=3\sin t+C$ $s(0)=3\sin 0+C$ $0=3\sin 0+C\Rightarrow C=0$ $s(t)=3\sin t$ $s\left(\frac{\pi}{6}\right)=3\sin(\frac{\pi}{6})=1.5\text{ m}$
b	$s(2\pi)-s(0)=3\sin(2\pi)-3\sin(0)=0\text{ m}$

c	$$\lvert v(t)\rvert = \lvert 3\cos t\rvert = \begin{cases} 3\cos t, 0 \le t < \frac{\pi}{2} \\ -3\cos t, \frac{\pi}{2} \le t \le \frac{3\pi}{2} \\ 3\cos t, \frac{3\pi}{2} < t \le 2\pi \end{cases}$$ $$\int_0^{2\pi} \lvert v(t)\rvert\, dx = \int_0^{\frac{\pi}{2}} 3\cos t\ dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -3\cos t\ dx + \int_{\frac{3\pi}{2}}^{2\pi} 3\cos t\ dx$$ $$= 3\sin t\Big\rvert_0^{\frac{\pi}{2}} - 3\sin t\Big\rvert_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + 3\sin t\Big\rvert_{\frac{3\pi}{2}}^{2\pi}$$ $$= (3-0) - (-3-3) + (0-(-3)) = 12\text{ m}$$
أتدرب وأحل المسائل صفحة 17	
1	$\int (e^{2x-3} - \sqrt{x})dx = \int \left(e^{2x-3} - x^{1/2}\right)dx = \frac{1}{2}e^{2x-3} - \frac{2}{3}x^{3/2} + C$
2	$\int \left(e^{0.5x} - \frac{3}{e^{0.5x}}\right)dx = \int (e^{0.5x} - 3e^{-0.5x})dx = 2e^{0.5x} + 6e^{-0.5x} + C$
3	$\int (4\sin 5x - 5\cos 4x)dx = -\frac{4}{5}\cos 5x - \frac{5}{4}\sin 4x + C$
4	$\int \left(3\sec x\tan x - \frac{2}{5x}\right)dx = 3\sec x - \frac{2}{5}\ln\lvert x\rvert + C$
5	$$\int \left(\sqrt{e^x} - \frac{1}{\sqrt{e^x}}\right)^2 dx = \int \left(e^x - 2 + \frac{1}{e^x}\right)dx$$ $$= \int (e^x - 2 + e^{-x})dx$$ $$= e^x - 2x - e^{-x} + C$$
6	$\int (\sin(5-3x) + 2 + 4x^2)dx = \frac{1}{3}\cos(5-3x) + 2x + \frac{4}{3}x^3 + C$
7	$\int (e^x + 1)^2 dx = \int (e^{2x} + 2e^x + 1)dx = \frac{1}{2}e^{2x} + 2e^x + x + C$
8	$$\int (e^{4-x} + \sin(4-x) + \cos(4-x))dx$$ $$= -e^{4-x} + \cos(4-x) - \sin(4-x) + C$$

9	$\int \frac{x^2-6}{2x}dx = \int \left(\frac{1}{2}x - \frac{3}{x}\right)dx$ $= \frac{1}{4}x^2 - 3\ln\lvert x\rvert + C$
10	$\int \left(3\csc^2(3x+2) + \frac{5}{x}\right)dx = -\cot(3x+2) + 5\ln\lvert x\rvert + C$
11	$\int \frac{e^x+1}{e^x}dx = \int (1+e^{-x})dx = x - e^{-x} + C$
12	$\int \frac{e^x}{e^x+4}dx = \ln\lvert e^x+4\rvert + C = \ln(e^x+4) + C$
13	$\int \frac{\cos 2x}{\sin x\cos x + 4}dx = \int \frac{\cos 2x}{\frac{1}{2}\sin 2x + 4}dx$ $= \ln\left\lvert\frac{1}{2}\sin 2x + 4\right\rvert + C = \ln\left(\frac{1}{2}\sin 2x + 4\right) + C$
14	$\int \frac{dx}{5-\frac{x}{3}} = -3\int \frac{-\frac{1}{3}}{5-\frac{x}{3}}dx$ $= -3\ln\left\lvert 5-\frac{x}{3}\right\rvert + C$
15	$\int \frac{1}{1-\sin x}dx = \int \frac{1}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}dx$ $= \int \frac{1+\sin x}{1-\sin^2 x}dx$ $= \int \frac{1+\sin x}{\cos^2 x}dx$ $= \int (\sec^2 x + \tan x\sec x)dx$ $= \tan x + \sec x + C$
16	$\int \sec^2 x\,(1+e^x\cos^2 x)dx = \int (\sec^2 x + e^x)dx$ $= \tan x + e^x + C$
17	$\int \left(\frac{2}{x} - 2^x\right)dx = 2\ln\lvert x\rvert - \frac{2^x}{\ln 2} + C$

18	$\int \sin 3x \cos 2x\, dx = \frac{1}{2}\int (\sin 5x + \sin x) dx$ $= -\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C$
19	$\int \frac{2x+3}{3x^2+9x-1} dx = \frac{1}{3}\int \frac{6x+9}{3x^2+9x-1} dx$ $= \frac{1}{3}\ln\lvert 3x^2+9x-1\rvert + C$
20	$\int \frac{x^2+x+1}{x^2+1} dx = \int \left(\frac{x^2+1}{x^2+1} + \frac{x}{x^2+1}\right) dx$ $= \int \left(1 + \frac{1}{2} \times \frac{2x}{x^2+1}\right) dx$ $= x + \frac{1}{2}\ln(x^2+1) + C$
21	$\int \left(\frac{1+\cos x}{\sin^2 x} + \sin^2 x \csc x\right) dx = \int (\csc^2 x + \cot x \csc x + \sin x) dx$ $= -\cot x - \csc x - \cos x + C$
22	$\int (\sec x + \tan x)^2 dx = \int (\sec^2 x + 2\sec x \tan x + \tan^2 x) dx$ $= \int (\sec^2 x + 2\sec x \tan x + \sec^2 x - 1) dx$ $= \int (2\sec^2 x + 2\sec x \tan x - 1) dx$ $= 2\tan x + 2\sec x - x + C$
23	$\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \ln(e^x + e^{-x}) + C$
24	$\int \frac{x^2}{x^3-3} dx = \frac{1}{3}\int \frac{3x^2}{x^3-3} dx = \frac{1}{3}\ln\lvert x^3-3\rvert + C$

25	$\int(9\cos^2x-\sin^2x-6\sin x\cos x)dx$ $=\int(9\cos^2x-(1-\cos^2x)-6\sin x\cos x)dx$ $=\int(10\cos^2x-1-6\sin x\cos x)dx$ $=\int\left(10\left(\frac{1+\cos 2x}{2}\right)-1-3\sin 2x\right)dx$ $=\int(5+5\cos 2x-1-3\sin 2x)dx=\int(4+5\cos 2x-3\sin 2x)dx$ $=4x+\frac{5}{2}\sin 2x+\frac{3}{2}\cos 2x+C$
26	$\int(\cos^4x-\sin^4x)dx=\int(\cos^2x-\sin^2x)(\cos^2x+\sin^2x)dx$ $=\int(\cos^2x-\sin^2x)dx$ $=\int\cos 2x\,dx$ $=\frac{1}{2}\sin 2x+C$
27	$\int_0^{\pi}2\cos\frac{1}{2}x\;dx=4\sin\frac{1}{2}x\Big\|_0^{\pi}=4\left(\sin\frac{\pi}{2}-\sin 0\right)=4$
28	$\|\sin x\|=\begin{cases}\sin x & ,0\le x\le\pi\\ -\sin x & ,\pi<x\le 2\pi\end{cases}$ $\int_0^{2\pi}\|\sin x\|\,dx=\int_0^{\pi}\sin x\;dx+\int_{\pi}^{2\pi}-\sin x\;dx$ $=-\cos x\|_0^{\pi}+\cos x\|_{\pi}^{2\pi}$ $=-(\cos\pi-\cos 0)+\cos 2\pi-\cos\pi$ $=-(-2)+1-(-1)=4$

29	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 3\tan^2 x\ dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 3(\sec^2 x - 1)\ dx$ $= 3(\tan x - x)\Big\|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ $= 3\left(\tan\frac{\pi}{3} - \frac{\pi}{3}\right) - 3\left(\tan\frac{\pi}{6} - \frac{\pi}{6}\right)$ $= 2\sqrt{3} - \frac{\pi}{2}$
30	$\int_1^e \frac{8x}{x^2+1}\ dx = 4\int_1^e \frac{2x}{x^2+1}\ dx$ $= 4\ln\|x^2+1\|\Big\|_1^e$ $= 4\ln(e^2+1) - 4\ln 2$ $= 4\ln\left(\frac{e^2+1}{2}\right)$
31	$\int_0^{\frac{\pi}{6}} \sin 3x \cos x\ dx = \frac{1}{2}\int_0^{\frac{\pi}{6}} (\sin 4x + \sin 2x)\ dx$ $= \left(-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x\right)\Big\|_0^{\frac{\pi}{6}}$ $= -\frac{1}{8}\cos\frac{4\pi}{6} - \frac{1}{4}\cos\frac{2\pi}{6} - (-\frac{1}{8}\cos 0 - \frac{1}{4}\cos 0)$ $= \frac{5}{16}$

32	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cot^2 x}{1+\cot^2 x}\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cot^2 x}{\csc^2 x}\,dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos^2 x}{\sin^2 \csc^2 x}\,dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos^2 x}{\sin^2 x(\frac{1}{\sin^2 x})}\,dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2 x\,dx$ $= \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1+\cos 2x)dx$ $= \frac{1}{2}\left(x+\frac{1}{2}\sin 2x\right)\Big\|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \frac{\pi}{6}+\frac{1}{4}\sin\frac{2\pi}{3} - (\frac{\pi}{8}+\frac{1}{4}\sin\frac{2\pi}{4})$ $= \frac{3\sqrt{3}+\pi-6}{24}$
33	$\int_0^3 (x-5^x)\,dx = \left(\frac{1}{2}x^2 - \frac{5^x}{\ln 5}\right)\Big\|_0^3$ $= \frac{9}{2} - \frac{125}{\ln 5} - (0 - \frac{1}{\ln 5}) = \frac{9}{2} - \frac{124}{\ln 5}$

34	$|x^2-4x+3| = \begin{cases} x^2-4x+3 & ,x<1 \\ -x^2+4x-3, & 1\le x\le 3 \\ x^2-4x+3 & ,x>3 \end{cases}$ $\int_0^4 |x^2-4x+3|\,dx$ $= \int_0^1 (x^2-4x+3)\,dx + \int_1^3 (-x^2+4x-3)\,dx + \int_3^4 (x^2-4x+3)\,dx$ $= \left(\frac{1}{3}x^3-2x^2+3x\right)\Big|_0^1 + \left(-\frac{1}{3}x^3+2x^2-3x\right)\Big|_1^3 + \left(\frac{1}{3}x^3-2x^2+3x\right)\Big|_3^4$ $= \frac{1}{3}-2+3-0+(-9+18-9)-\left(-\frac{1}{3}+2-3\right)+\frac{64}{3}-32+12$ $-(9-18+9)=4$
35	$|x-3| = \begin{cases} 3-x & ,x\le 3 \\ x-3 & ,x>3 \end{cases}$ $\int_1^4 (3-|x-3|)\,dx = \int_1^3 (3-(3-x))\,dx + \int_3^4 (3-(x-3))\,dx$ $= \int_1^3 x\,dx + \int_3^4 (6-x)\,dx$ $= \frac{1}{2}x^2\Big|_1^3 + \left(6x-\frac{1}{2}x^2\right)\Big|_3^4$ $= \frac{9}{2}-\frac{1}{2}+24-8-(18-\frac{9}{2})$ $= \frac{13}{2}$

36	$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} (x^2+4)\,dx + \int_{0}^{1} (4-x)\,dx$ $= \left(\frac{1}{3}x^3 + 4x\right)\Big\|_{-1}^{0} + \left(4x - \frac{1}{2}x^2\right)\Big\|_{0}^{1}$ $= 0 - \left(-\frac{1}{3} - 4\right) + 4 - \frac{1}{2} - 0$ $= \frac{47}{6}$
37	$A = \int_{2}^{4} (e^{0.5x} - 2)\,dx = (2e^{0.5x} - 2x)\big\|_{2}^{4}$ $= 2e^2 - 8 - (2e - 4)$ $= 2e^2 - 2e - 4$
38	$\int_{a}^{3a} \frac{2x+1}{x}\,dx = \int_{a}^{3a} 2 + \frac{1}{x}\,dx$ $= (2x + \ln\|x\|)\big\|_{a}^{3a}$ $= 6a + \ln 3a - 2a - \ln a$ $= 4a + \ln 3$ $\Rightarrow 4a + \ln 3 = \ln 12$ $\Rightarrow 4a = \ln 12 - \ln 3$ $4a = \ln \frac{12}{3}$ $\Rightarrow a = \frac{1}{4}\ln 4$
39	$\int_{0}^{a} \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\int_{0}^{a} \frac{2x}{x^2+a^2}\,dx$ $= \frac{1}{2}\ln(x^2+a^2)\Big\|_{0}^{a}$ $= \frac{1}{2}(\ln(2a^2) - \ln(a^2)) = \frac{1}{2}\ln 2 = \ln\sqrt{2}$

40	$A=\int_1^a \frac{4}{x}\,dx = 4\ln\lvert x\rvert\rvert_1^a = 4\ln a - 4\ln 1 = 4\ln a$ $\Rightarrow 4\ln a = 10 \Rightarrow \ln a = \frac{5}{2} \Rightarrow a = e^{\frac{5}{2}}$
41	$f(x)=\int \cos\left(\frac{1}{2}x+\pi\right)dx = 2\sin\left(\frac{1}{2}x+\pi\right)+C$ $f(\pi)=2\sin(\frac{1}{2}\pi+\pi)+C$ $3=2\sin\frac{3\pi}{2}+C$ $3=-2+C \quad \Rightarrow C=5$ $\Rightarrow f(x)=2\sin(\frac{1}{2}x+\pi)+5$ $\Rightarrow f(0)=2\sin\pi+5=5$
42	$y=\int \sin\left(\frac{\pi}{2}-2x\right)dx = \frac{-\cos\left(\frac{\pi}{2}-2x\right)}{-2}+C=\frac{1}{2}\cos\left(\frac{\pi}{2}-2x\right)+C$ $y\rvert_{x=\frac{\pi}{4}}=\frac{1}{2}\cos(\frac{\pi}{2}-\frac{\pi}{2})+C$ $1=\frac{1}{2}+C \quad \Rightarrow C=\frac{1}{2}$ $\Rightarrow y=\frac{1}{2}\cos\left(\frac{\pi}{2}-2x\right)+\frac{1}{2}$ $\Rightarrow y=\frac{1}{2}\sin 2x+\frac{1}{2}$ $\Rightarrow y=\frac{1+\sin 2x}{2}$

43	$y = \int (e^{2x} - 2e^{-x})\, dx = \frac{1}{2}e^{2x} + 2e^{-x} + C$ $y\|_{x=0} = \frac{1}{2} + 2 + C$ $1 = \frac{5}{2} + C \Rightarrow C = -\frac{3}{2}$ $\Rightarrow y = \frac{1}{2}e^{2x} + 2e^{-x} - \frac{3}{2}$
44	$\int_{\frac{\pi}{9}}^{\pi} (9 + \sin 3x)\, dx = \left(9x - \frac{1}{3}\cos 3x\right)\Big\|_{\frac{\pi}{9}}^{\pi}$ $= 9\pi - \frac{1}{3}\cos 3\pi - \pi + \frac{1}{3}\cos\frac{\pi}{3}$ $= 8\pi + \frac{1}{3} + \frac{1}{6}$ $= 8\pi + \frac{1}{2}$ $\Rightarrow 8\pi + \frac{1}{2} = a\pi + b$ ونظرًا لأن a و b نسبيّان، فلا يوجد حل لهذه المعادلة سوى أن يكون: $a = 8 , b = \frac{1}{2}$
45	$f(x) = \int \cos^2 x\, dx = \frac{1}{2}\int (1 + \cos 2x)\, dx = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C$ $f(0) = \frac{1}{2}\left(0 + \frac{1}{2}\sin 0\right) + C$ $0 = 0 + C \Rightarrow C = 0$ $f(x) = \frac{1}{2}x + \frac{1}{4}\sin 2x$

46	$s(t) = \int e^{-2t}\,dt = -\frac{1}{2}e^{-2t} + C$ $s(0) = -\frac{1}{2} + C = 3 \Rightarrow C = \frac{7}{2}$ $3 = -\frac{1}{2} + C \Rightarrow C = \frac{7}{2}$ $s(t) = -\frac{1}{2}e^{-2t} + \frac{7}{2}$
47	$s(100) = -\frac{1}{2}e^{-200} + \frac{7}{2} \approx 3.5\,m$
48	$P(t) = \int -0.51e^{-0.03t}\,dt = \frac{-0.51}{-0.03}e^{-0.03t} + C = 17e^{-0.03t} + C$ $P(0) = 17 + C$ $500 = 17 + C \Rightarrow C = 483$ $P(t) = 17e^{-0.03t} + 483$
49	$P(10) = 17e^{-0.3} + 483 \approx 496$
50	$P(t) = \int (0.15 - 0.9e^{0.006t})\,dt$ $= 0.15t - \frac{0.9}{0.006}e^{0.006t} + C$ $= 0.15t - 150e^{0.006t} + C$ $P(0) = -150 + C$ $30 = -150 + C \Rightarrow C = 180$ $P(t) = 0.15t - 150e^{0.006t} + 180$
51	$P(10) = 1.5 - 150e^{0.06} + 180 \approx 22.2\ cm^3$

52	$\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$ $A = \int_0^{\pi} \sin x \, dx + (-\int_{\pi}^{2\pi} \sin x \, dx) = (-\cos x)\|_0^{\pi} + (\cos x)\|_{\pi}^{2\pi}$ $= -\cos \pi + \cos 0 + \cos 2\pi - \cos \pi = 4$ ملحوظة: يمكن الاستفادة من التماثل وإيجاد المساحة المطلوبة كما يأتي: $A = 2\int_0^{\pi} \sin x \, dx = 2(-\cos x)\|_0^{\pi} = 2(-\cos \pi + \cos 0) = 2(2) = 4$
53	$\sin 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ $A = \int_0^{\frac{\pi}{2}} \sin 2x \, dx + (-\int_{\frac{\pi}{2}}^{\pi} \sin 2x \, dx) + \int_{\pi}^{\frac{3\pi}{2}} \sin 2x \, dx$ $+ (-\int_{\frac{3\pi}{2}}^{2\pi} \sin 2x \, dx)$ والأسهل هو الاستفادة من التماثل وإيجاد المساحة المطلوبة كما يأتي: $A = 4\int_0^{\frac{\pi}{2}} \sin 2x \, dx = -2\cos 2x\|_0^{\frac{\pi}{2}} = -2(-1-1) = 4$
54	نقسم البسط والمقام على $\cos x$ $\int \frac{\sec x}{\sin x - \cos x} dx = \int \frac{\frac{\sec x}{\cos x}}{\left(\frac{\sin x}{\cos x} - 1\right)} dx$ $= \int \frac{\sec^2 x}{(\tan x - 1)} dx$ $= \ln\|\tan x - 1\| + C$

55	نضرب البسط والمقام في $\csc x$ $\int \frac{\cot x}{2+\sin x}\,dx = \int \frac{\cot x \csc x}{2\csc x + 1}\,dx$ $= -\frac{1}{2}\int \frac{-2\cot x \csc x}{2\csc x + 1}\,dx$ $= -\frac{1}{2}\ln\lvert 2\csc x + 1\rvert + C$ $= -\frac{1}{2}\ln\lvert 2\csc x + 1\rvert + C$
56	$\int \frac{1}{x\ln x^3}\,dx = \int \frac{1}{3x\ln x}\,dx = \frac{1}{3}\int \frac{\frac{1}{x}}{\ln x}\,dx = \frac{1}{3}\ln\lvert\ln x\rvert + C$
57	$\int_1^a \left(\frac{1}{x} - \frac{1}{2x+3}\right)dx = \left(\ln\lvert x\rvert - \frac{1}{2}\ln\lvert 2x+3\rvert\right)\Big\rvert_1^a$ $= \left(\ln a - \frac{1}{2}\ln(2a+3)\right) - \left(-\frac{1}{2}\ln 5\right)$ $= \ln a - \frac{1}{2}\ln(2a+3) + \frac{1}{2}\ln 5$ $= \ln\frac{a}{\sqrt{2a+3}} + \frac{1}{2}\ln 5$ $\Rightarrow \ln\frac{a}{\sqrt{2a+3}} + \frac{1}{2}\ln 5 = 0.5\ln 5$ $\Rightarrow \ln\frac{a}{\sqrt{2a+3}} = 0$ $\Rightarrow \frac{a}{\sqrt{2a+3}} = 1$ $\Rightarrow a = \sqrt{2a+3}$ $\Rightarrow a^2 = 2a+3$ $\Rightarrow a^2 - 2a - 3 = 0$ $\Rightarrow (a-3)(a+1) = 0$ $\Rightarrow a = 3 \quad ,a = -1$ مرفوضة لأن $a > 0$

58	طريقة أولى: $\int_0^{\frac{\pi}{4}} \cos x \cos 3x\ dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} (\cos 4x + \cos 2x)\ dx$ $= \left(\frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x\right)\Big\|_0^{\frac{\pi}{4}}$ $= \left(\frac{1}{8}\sin \pi + \frac{1}{4}\sin \frac{\pi}{2}\right) - (0+0) = \frac{1}{4} \ldots\ldots\ldots\ldots (1)$ $\int_0^{\frac{\pi}{4}} \sin x \sin 3x\ dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} (\cos 2x - \cos 4x)\ dx$ $= \left(\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x\right)\Big\|_0^{\frac{\pi}{4}}$ $= \left(\frac{1}{4}\sin \frac{\pi}{2} - \frac{1}{8}\sin \pi\right) - (0-0) = \frac{1}{4} \ldots\ldots\ldots\ldots (2)$ $\Rightarrow \int_0^{\frac{\pi}{4}} \cos x \cos 3x\ dx - \int_0^{\frac{\pi}{4}} \sin x \sin 3x\ dx = \frac{1}{4} - \frac{1}{4} = 0$ طريقة ثانية: $\int_0^{\frac{\pi}{4}} \cos x \cos 3x\ dx - \int_0^{\frac{\pi}{4}} \sin x \sin 3x\ dx$ $= \int_0^{\frac{\pi}{4}} (\cos x \cos 3x - \sin x \sin 3x)\ dx = \int_0^{\frac{\pi}{4}} \cos(x+3x)\ dx$ $= \int_0^{\frac{\pi}{4}} \cos 4x\ dx = \frac{1}{4}\sin 4x\Big\|_0^{\frac{\pi}{4}} = \frac{1}{4}(\sin \pi - \sin 0) = 0$

59	$\int_{\frac{\pi}{4k}}^{\frac{\pi}{3k}} (1-\pi\sin kx)\,dx = \left(x+\frac{\pi}{k}\cos kx\right)\Big\vert_{\frac{\pi}{4k}}^{\frac{\pi}{3k}}$ $=\frac{\pi}{3k}+\frac{\pi}{k}\cos\frac{\pi}{3}-\frac{\pi}{4k}-\frac{\pi}{k}\cos\frac{\pi}{4}$ $=\frac{\pi}{k}\left(\frac{1}{3}+\frac{1}{2}-\frac{1}{4}-\frac{\sqrt{2}}{2}\right)=\frac{\pi}{12k}(7-6\sqrt{2})$ $\Rightarrow \frac{\pi}{12k}(7-6\sqrt{2})=\pi(7-6\sqrt{2})$ $\Rightarrow k=\frac{1}{12}$
60	$v(t)=\begin{cases} 2t+4 , & 0\le t\le 6 \\ 16t-t^2-44 , & 6<t\le 10\end{cases}$ $s(t)=\int v(t)\,dt$ عندما $0\le t\le 6$: $s(t)=\int(2t+4)\,dt=t^2+4t+C_1$ $s(0)=0\Rightarrow C_1=0$ $\Rightarrow s(t)=t^2+4t ,0\le t\le 6$ $s(5)=25+20=45\text{ m}$

61	عندما $6 < t \le 10$: $s(t) = \int (16t - t^2 - 44)\, dt = 8t^2 - \frac{1}{3}t^3 - 44t + C_2$ لإيجاد قيمة C_2 نستعمل موقع الجسم عند t = 6 موقعًا ابتدائيًا بالنسبة للفترة [6, 10] $s(6) = 8(6)^2 - \frac{1}{3}(6)^3 - 44(6) + C_2$ ونحسب s(6) من اقتران الموقع الذي وجدناه في السؤال السابق بالنسبة للفترة [0, 6] $s(t) = t^2 + 4t \;, 0 \le t \le 6$ $s(6) = 6^2 + 4(6) = 60$ $60 = 8(6)^2 - \frac{1}{3}(6)^3 - 44(6) + C_2$ $60 = -48 + C_2 \Rightarrow C_2 = 108$ $\Rightarrow s(t) = 8t^2 - \frac{1}{3}t^3 - 44t + 108 \;, 6 < t \le 10$ $s(9) = 117 \text{ m}$
62	$A = \int_0^{45} \frac{1}{x+3}\, dx = \ln\lvert x+3\rvert \big\rvert_0^{45}$ $= \ln 48 - \ln 3 = \ln 16$ $\frac{1}{2}A = \int_0^{k} \frac{1}{x+3}\, dx = \ln\lvert x+3\rvert \big\rvert_0^{k}$ $= \ln(k+3) - \ln 3$ $= \ln\frac{k+3}{3}$ $\Rightarrow \frac{1}{2}\ln 16 = \ln\frac{k+3}{3}$ $\ln 16^{1/2} = \ln\frac{k+3}{3}$ $\Rightarrow 4 = \frac{k+3}{3} \Rightarrow k = 9$