



اختبار نهاية الوحدة صفحة 72

1	c
2	b
3	d
4	d
5	c
6	a
7	d
8	$f(x) = e^x(x + x\sqrt{x})$ $f'(x) = (e^x) \left(1 + (x) \left(\frac{1}{2\sqrt{x}} \right) + (\sqrt{x})(1) \right) + (x + x\sqrt{x})(e^x)$ $= e^x \left(1 + \frac{3}{2}\sqrt{x} + x + x\sqrt{x} \right)$
9	$f(x) = \frac{x}{\tan x}$ $f'(x) = \frac{(\tan x)(1) - (x)(\sec^2 x)}{\tan^2 x} = \frac{\tan x - x \sec^2 x}{\tan^2 x}$
10	$f(x) = \frac{1}{x} - 12 \sec x$ $f'(x) = \frac{-1}{x^2} - 12 \sec x \tan x$
11	$f(x) = \frac{e^x}{\ln x}$ $f'(x) = \frac{(\ln x)(e^x) - (e^x) \left(\frac{1}{x} \right)}{\ln^2 x} = \frac{e^x(x \ln x - 1)}{x \ln^2 x}$
12	$f(x) = \frac{\ln x}{x^4}$ $f'(x) = \frac{(x^4) \left(\frac{1}{x} \right) - (\ln x)(4x^3)}{x^8} = \frac{1 - 4 \ln x}{x^5}$





13	$f(x) = 5^{2-x}$ $\ln f(x) = \ln 5^{2-x}$ $\ln f(x) = (2-x) \ln 5$ $\frac{f'(x)}{f(x)} = -\ln 5$ $f'(x) = -(\ln 5)f(x) = -(\ln 5)(5^{2-x})$
14	$f(x) = 10 \sin 0.5x$ $f'(x) = 5 \cos 0.5x$
15	$f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)^3 \left(x + \frac{1}{x}\right)^2$ $f'(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)^3 \left(2\left(x + \frac{1}{x}\right)\left(1 - \frac{1}{x^2}\right)\right) + \left(x + \frac{1}{x}\right)^2 \left(3\left(\frac{1}{x} + \frac{1}{x^2}\right)^2 \left(-\frac{1}{x^2} - \frac{2}{x^3}\right)\right)$ $= \left(\frac{1}{x} + \frac{1}{x^2}\right)^2 \left(x + \frac{1}{x}\right) \left(2\left(\frac{1}{x} + \frac{1}{x^2}\right)\left(1 - \frac{1}{x^2}\right) + 3\left(x + \frac{1}{x}\right)\left(-\frac{1}{x^2} - \frac{2}{x^3}\right)\right)$ $= \left(\frac{1}{x} + \frac{1}{x^2}\right)^2 \left(x + \frac{1}{x}\right) \left(\frac{2}{x} - \frac{2}{x^3} + \frac{2}{x^2} - \frac{2}{x^4} - \frac{3}{x} - \frac{6}{x^2} - \frac{3}{x^3} - \frac{6}{x^4}\right)$ $= -\left(\frac{1}{x} + \frac{1}{x^2}\right)^2 \left(x + \frac{1}{x}\right) \left(\frac{8 + 5x + 4x^2 + x^3}{x^4}\right)$
16	$f(x) = e^{-1.5x} \cos x^2$ $f'(x) = (e^{-1.5x})(-2x \sin x^2) + (\cos x^2)(-1.5e^{-1.5x})$ $= -e^{-1.5x}(2x \sin x^2 + \cos x^2)$
17	$(fg)'(2) = f(2)g'(2) + g(2)f'(2)$ $= 3 \times 2 + 1 \times -4 = 2$
18	$\left(\frac{f}{g}\right)'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{g^2(2)} = \frac{1 \times -4 - 3 \times 2}{(1)^2} = -10$
19	$(3f - 4fg)'(2) = 3f'(2) - 4(fg)'(2) = 3(-4) - 4(2) = -20$
20	$f(x) = x^7 \ln x$ $f'(x) = (x^7)\left(\frac{1}{x}\right) + (\ln x)(7x^6) = x^6 + 7x^6 \ln x$ $f''(x) = 6x^5 + (7x^6)\left(\frac{1}{x}\right) + (\ln x)(42x^5) = 13x^5 + 42x^5 \ln x$



21	$f(x) = \frac{\cos x}{x}$ $f'(x) = \frac{(x)(-\sin x) - (\cos x)(1)}{x^2} = \frac{-\sin x}{x} - \frac{\cos x}{x^2}$ $f''(x) = \frac{x \cos x - \sin x}{x^2} - \frac{-x^2 \sin x - 2x \cos x}{x^4}$ $= \frac{-x^3 \cos x + 2x^2 \sin x + 2x \cos x}{x^4}$ $= \frac{-x^2 \cos x + 2x \sin x + 2 \cos x}{x^3}$
22	$f(x) = \frac{x}{1 + \sqrt{x}}$ $f'(x) = \frac{(1 + \sqrt{x})(1) - (x)\left(\frac{1}{2\sqrt{x}}\right)}{(1 + \sqrt{x})^2} = \frac{1 + \frac{1}{2}\sqrt{x}}{(1 + \sqrt{x})^2}$ $f''(x) = \frac{(1 + \sqrt{x})^2 \left(\frac{1}{4\sqrt{x}}\right) - \left(1 + \frac{1}{2}\sqrt{x}\right) \left(\frac{1}{\sqrt{x}}(1 + \sqrt{x})\right)}{(1 + \sqrt{x})^4}$ $= \frac{-3 - \sqrt{x}}{4\sqrt{x}(1 + \sqrt{x})^3}$
23	$f(x) = \frac{1 - x^2}{1 + x^2}$ $f'(x) = \frac{(1 + x^2)(-2x) - (1 - x^2)(2x)}{(1 + x^2)^2} = \frac{-4x}{(1 + x^2)^2}$ $f''(x) = \frac{(1 + x^2)^2(-4) - (-4x)(2 \times 2x(1 + x^2))}{(1 + x^2)^4}$ $= \frac{12x^2 - 4}{(1 + x^2)^3}$





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$$f(x) = \frac{x^2}{1+x}$$

$$f(1) = \frac{1}{1+1} = \frac{1}{2} \rightarrow \left(1, \frac{1}{2}\right)$$

$$f'(x) = \frac{(1+x)(2x) - (x^2)(1)}{(1+x)^2} = \frac{x^2 + 2x}{(1+x)^2}$$

$$f'(1) = \frac{3}{4}$$

$$y - \frac{1}{2} = \frac{3}{4}(x - 1) \rightarrow y = \frac{3}{4}x - \frac{1}{4}$$

نقطة التماس:

ميل المماس:

معادلة المماس:

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$$f(x) = \frac{x^2}{\cos x}$$

$$f\left(\frac{\pi}{4}\right) = \frac{\left(\frac{\pi}{4}\right)^2}{\cos\left(\frac{\pi}{4}\right)} = \frac{\frac{\pi^2}{16}}{\frac{1}{\sqrt{2}}} = \frac{\pi^2\sqrt{2}}{16} \rightarrow \left(\frac{\pi}{4}, \frac{\pi^2\sqrt{2}}{16}\right)$$

$$f'(x) = \frac{(\cos x)(2x) - (x^2)(-\sin x)}{(\cos x)^2}$$

$$f'\left(\frac{\pi}{4}\right) = \frac{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\pi}{2}\right) - \left(\frac{\pi^2}{16}\right)\left(-\frac{1}{\sqrt{2}}\right)}{\frac{1}{2}} = \frac{8\pi + \pi^2}{8\sqrt{2}}$$

$$y - \frac{\pi^2\sqrt{2}}{16} = \frac{8\pi + \pi^2}{8\sqrt{2}}\left(x - \frac{\pi}{4}\right)$$

نقطة التماس:

ميل المماس:

معادلة المماس:

منهجي

منعة التعليم الهادف





26	$f(x) = \ln(x + 5)$ <p>نقطة التماس:</p> $f(0) = \ln(0 + 5) = \ln(5) \rightarrow (0, \ln 5)$ <p>ميل المماس:</p> $f'(x) = \frac{1}{x + 5}$ $f'(0) = \frac{1}{5}$ <p>معادلة المماس:</p> $y - \ln 5 = \frac{1}{5}(x - 0) \rightarrow y = \frac{1}{5}x + \ln 5$
27	$f(x) = \sin x + \sin 3x$ <p>نقطة التماس:</p> $f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} + \sin \frac{3\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \rightarrow \left(\frac{\pi}{4}, \sqrt{2}\right)$ <p>ميل المماس:</p> $f'(x) = \cos x + 3 \cos 3x$ $f'\left(\frac{\pi}{4}\right) = \cos \frac{\pi}{4} + 3 \cos \frac{3\pi}{4} = \frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}} = -\sqrt{2}$ <p>معادلة المماس:</p> $y - \sqrt{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right) \rightarrow y = -\sqrt{2}x + \sqrt{2}\left(\frac{\pi}{4} + 1\right)$
28	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{2t}$ <p>ميل المماس:</p> $m = \left.\frac{dy}{dx}\right _{t=4} = \frac{1}{8}$ <p>نقطة التماس:</p> $x = (4)^2 = 16, y = 4 + 2 = 6 \rightarrow (16, 6)$ <p>معادلة المماس:</p> $y - 6 = \frac{1}{8}(x - 16) \rightarrow y = \frac{1}{8}x + 4$



<p>29</p>	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3 \cos t}{-4 \sin t} = -\frac{3}{4} \cot t$ <p>ميل المماس:</p> $m = \left. \frac{dy}{dx} \right _{t=\frac{\pi}{4}} = -\frac{3}{4} \cot \frac{\pi}{4} = -\frac{3}{4}$ <p>نقطة التماس:</p> $x = 4 \cos \frac{\pi}{4} = 2\sqrt{2}, \quad y = 3 \sin \frac{\pi}{4} = \frac{3\sqrt{2}}{2} \rightarrow \left(2\sqrt{2}, \frac{3\sqrt{2}}{2} \right)$ <p>معادلة المماس:</p> $y - \frac{3\sqrt{2}}{2} = -\frac{3}{4}(x - 2\sqrt{2}) \rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$
<p>30</p>	<p>ميل المماس:</p> $y = x \ln x$ $f'(x) = (x) \left(\frac{1}{x} \right) + (\ln x)(1) = 1 + \ln x$ $f'(1) = 1 + \ln 1 = 1$ <p>معادلة المماس:</p> $y - 0 = 1(x - 1) \rightarrow y = x - 1$
<p>31</p>	$f'(x) = 2 \rightarrow 1 + \ln x = 2$ $\rightarrow \ln x = 1$ $\rightarrow x = e \rightarrow y = e \ln e = e$ <p>النقطة المطلوبة هي (e, e)</p>
<p>32</p>	$x(x + y) = 2y^2 \rightarrow x^2 + xy = 2y^2$ $\rightarrow 2x + x \frac{dy}{dx} + y = 4y \frac{dy}{dx}$ $\rightarrow \frac{dy}{dx} = \frac{2x + y}{4y - x}$



33	$x = \frac{2y}{x^2 - y} \rightarrow x^3 - xy = 2y$ $\rightarrow 3x^2 - x \frac{dy}{dx} - y = 2 \frac{dy}{dx}$ $\rightarrow \frac{dy}{dx} = \frac{3x^2 - y}{x + 2}$
34	$y \cos x = x^2 + y^2 \rightarrow -y \sin x + \frac{dy}{dx} \cos x = 2x + 2y \frac{dy}{dx}$ $\rightarrow \frac{dy}{dx} = \frac{2x + y \sin x}{-2y + \cos x}$
35	$2xe^y + ye^x = 3 \rightarrow 2xe^y \frac{dy}{dx} + 2e^y + ye^x + e^x \frac{dy}{dx} = 0$ $\rightarrow \frac{dy}{dx} = -\frac{2e^y + ye^x}{2xe^y + e^x}$
36	$y^2 = \frac{x^3}{2-x}$ $2y \frac{dy}{dx} = \frac{(2-x)(3x^2) - (x^3)(-1)}{(2-x)^2}$ $2(-1) \frac{dy}{dx} = \frac{(2-1)(3) - (1)(-1)}{(2-1)^2} \rightarrow \frac{dy}{dx} = -2$ <p>ميل المماس:</p> $m = -2$ <p>ميل العمودي على المماس:</p> $m = \frac{-1}{-2} = \frac{1}{2}$ <p>معادلة العمودي على المماس:</p> $y + 1 = \frac{1}{2}(x - 1) \rightarrow y = \frac{1}{2}x - \frac{3}{2}$





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$$\begin{aligned}y &= \frac{(x+1)(x-2)}{(x-1)(x+2)} \\ \ln y &= \ln \frac{(x+1)(x-2)}{(x-1)(x+2)} \\ &= \ln(x+1) + \ln(x-2) - \ln(x-1) - \ln(x+2) \\ \frac{dy}{dx} &= \frac{1}{x+1} + \frac{1}{x-2} - \frac{1}{x-1} - \frac{1}{x+2} \\ \frac{dy}{dx} &= \left(\frac{1}{x+1} + \frac{1}{x-2} - \frac{1}{x-1} - \frac{1}{x+2} \right) y \\ &= \left(\frac{1}{x+1} + \frac{1}{x-2} - \frac{1}{x-1} - \frac{1}{x+2} \right) \frac{(x+1)(x-2)}{(x-1)(x+2)} \\ &= \left(\frac{-2}{x^2-1} + \frac{4}{x^2-4} \right) \frac{(x+1)(x-2)}{(x-1)(x+2)} \\ &= \left(\frac{2x^2+4}{(x^2-1)(x^2-4)} \right) \frac{(x+1)(x-2)}{(x-1)(x+2)} \\ &= \frac{2x^2+4}{(x-1)^2(x+2)^2}\end{aligned}$$

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$$\begin{aligned}y &= x^{\ln x} \\ \ln y &= \ln x^{\ln x} \\ &= (\ln x)(\ln x) = (\ln x)^2 \\ \frac{dy}{dx} &= 2(\ln x) \times \frac{1}{x} \\ \frac{dy}{dx} &= \left(\frac{2 \ln x}{x} \right) y \\ &= \left(\frac{2 \ln x}{x} \right) x^{\ln x}\end{aligned}$$





39	$x^2 + 3xy + y^2 = x + 3y$ $2x + 3x \frac{dy}{dx} + 3y + 2y \frac{dy}{dx} = 1 + 3 \frac{dy}{dx}$ $4 + 6 \frac{dy}{dx} - 3 - 2 \frac{dy}{dx} = 1 + 3 \frac{dy}{dx}$ $\frac{dy}{dx} \Big _{(2,-1)} = 0$ $y + 1 = 0(x - 2) \rightarrow y = -1$	ميل المماس عند $(2, -1)$: معادلة المماس:
40	$x^2 e^y = 1$ $x^2 e^y \frac{dy}{dx} + 2x e^y = 0$ $\frac{dy}{dx} + 2 = 0 \rightarrow \frac{dy}{dx} \Big _{(1,0)} = -2$ $y - 0 = -2(x - 1) \rightarrow y = -2x + 2$	ميل المماس: معادلة المماس:
41	$p'(1) = f(1)g'(1) + g(1)f'(1) = 2 \times 1 + 3 \times -2 = -4$	
42	$p'(4) = f(4)g'(4) + g(4)f'(4) = 1 \times 0 + 8 \times 0.5 = 4$	
43	$q'(7) = \frac{g(7)f'(7) - f(7)g'(7)}{(g(7))^2} = \frac{4 \times 2 - 4 \times -1}{(4)^2} = \frac{3}{4}$	
44	$R(t) = 200(0.9)^t$ $\frac{dR}{dt} = 200(0.9)^t \ln 0.9$ $\frac{dR}{dt} \Big _{t=2} = 200(0.9)^2 \ln 0.9 \approx -17.1 \text{ g/day}$	
45	$s(t) = 10 + \frac{1}{4} \sin(10\pi t)$ $v(t) = \frac{5\pi}{2} \cos(10\pi t)$ $a(t) = -25\pi^2 \sin(10\pi t)$	